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Infinitely many periodic solutions for discrete second order Hamiltonian systems with oscillating potential

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Abstract

In this article we obtain two sequence of infinitely many periodic solutions for discrete second order Hamiltonian systems with an oscillating potential. One sequence of solutions are local minimizers of the functional corresponding to the system, the other sequence are minimax type critical points of the functional.

1 Introduction

Discrete problems arise in the study of combinatorial analysis, quantum physics, chemical reactions, population dynamics, and so forth. Besides, they are also natural consequences of the discretization of continuous problems. On the other hand, the critical point theory has been a powerful tool in dealing with the existence and multiplicity results. Thus, discrete problems have been studied by many scholars via critical point theory. For example, Pankov and Zakharchenko used a variational method known as Nehari manifolds and a discrete version of the Lions concentration-compactness principle in [1] to establish existence results of nontrivial standing wave solutions for discrete nonlinear Schrödinger equation. Lately in [2], Pankov and Rothos employed Nehari manifolds approach and the Mountain Pass argument to demonstrate the existence of solutions in the discrete nonlinear Schrödinger equation with saturable nonlinearity. While for discrete Hamiltonian systems, Yu and Guo established a variational structure and introduced variational technique to the study of periodic solutions in [3-5].

The aim of this article is to apply the critical point theory to deal with the problem of infinitely multiplicity of periodic solutions for the following discrete second order Hamiltonian systems:

$$\begin{cases} \Delta^2 u(t-1) + \nabla F(t, u(t)) = 0, & t \in \mathbf{Z}[1, T] \\ u(0) = u(T), \end{cases} \tag{1}$$

where $\Delta u(t) := u(t+1) - u(t)$, $\Delta^2 u(t) = \Delta(\Delta u(t))$, and $\nabla F(t, x)$ denotes the gradient of F with respect to the second variable. Systems (1) can be considered as a discrete analog of the following Hamiltonian systems:

$$\begin{cases} -\ddot{u}(t) = \nabla F(u(t)), \\ u(0) - u(T) = \dot{u}(0) - \dot{u}(T) = 0, \end{cases}$$

where $\ddot{u}(t)$ denotes the second derivative of u with respect to t. Throughout of this article F will be called potential function, $(\cdot, \cdot)$ and $|\cdot|$ denote the inner product and norm in $\mathbf{R}^N$ respectively. A basic assumption we make on F is that F satisfies the following sublinear condition:

(F_α) $F(t, x) \in C^1(\mathbf{R}^N, \mathbf{R})$ for any $t \in \mathbf{Z}[0, T]$ and F is T-periodic in the first variable. Moreover there exist $f, g : \mathbf{Z}[0, T] \to \mathbf{R}^+$ and $0 < \alpha < 1$ such that

$$\left|\nabla F(t,x)\right| \le f(t)|x|^\alpha + g(t) \quad \text{for any } t \in \mathbf{Z}[0,T] \text{ and } x \in \mathbf{R}^N.$$

After the initial work of Yu and Guo, there appeared many results about Hamiltonian systems, such as [6-9]. Among these articles, the results in [5,6] have a close relation with the one in this article: they also considered (1) with sublinear or subquadratic potential. Especially, an existence result was obtained under the sublinear condition (F_α) and a partially coercive assumption:

$$|x|^{-2\alpha} \sum_{t=0}^{T} F(t,x) \to \infty \quad \text{as } |x| \to \infty \quad \text{for all } t \in \mathbf{Z}[0,T]. \tag{2}$$

But until now, no multiplicity results are obtained under subquadratic or sublinear condition.

On the other hand, the multiplicity problem was considered in [3,7-9]. When one study the multiplicity problem, an effective method is index theory. The index measure the size of subset which is invariant under some group action, such as Z_p action (for explicit definition, see [10]). If the functional is also invariant under this group action, The multiplicity of critical points can be obtained form the multiplicity of index. Guo and Yu [7] used the above mentioned Z_p index theory to show that there are at least T - 1 distinct Z_T-orbits for (1) when the potential function is autonomous and superquadratic, and at least $2(T$ - 1) distinct Z_T-orbits when moreover the potential function is even. They also obtained a result about the lower bounds for the number of T-periodic solutions for the asymptotically linear potential case. Their approach was based on Z_p index theory introduced in [10], so the autonomous condition is essential. However, infinitely many kinds of results can not be obtained form this method, since the index of the whole space if finite. For superquadratic system (1) where the potential may depend on time, Guo et al. obtained at least two nontrivial solutions in [3,9]. Later Xue and Tang obtained the same result under a more general superquadratic condition in [8].

Until now, as the authors knows there are no results of infinitely many kinds appeared for system (1). In this article, we are going to give some sufficient conditions to ensure (1) has infinitely many periodic solutions. Roughly speaking, instead of coercive assumption (2), we suppose F has a suitable oscillating behavior at infinity:

$$\limsup_{r\to\infty} \inf_{x\in\mathbf{R}^N, |x|=r} \sum_{t=0}^{T} F(t,x) = +\infty, \tag{3}$$

$$\liminf_{R\to\infty} \sup_{x\in\mathbf{R}^N, |x|=R} |x|^{-2\alpha} \sum_{t=0}^{T} F(t,x) = -\infty. \tag{4}$$

Then we obtain two sequence of infinitely many periodic solutions by minimax methods. One sequence of solutions is local minimizer of the functional ϕ corresponding to system (1), and the other are minimax type critical points of ϕ. The explicit form of ϕ will be given in Section 2. The idea in this article was inspired by Habets et al. [11] and Zhang and Tang [12], where Dirichlet type and periodic type boundary value problem for continuous systems were studied.

The following are main results:

Theorem 1.1 *Assume that F satisfies* (F_α), (3) *and* (4). *Then*

(*a*) *there exists a sequence* $\{u_n\}$ *of solutions of* (1) *such that* $\{u_n\}$ *is a critical point of* ϕ *and* $\lim_{n\to\infty} \phi(u_n) = +\infty$;

(*b*) *there exists a sequence* $\{u_n^*\}$ *of solutions of* (1) *such that* $\{u_n^*\}$ *is a local minimum of* ϕ *and* $\lim_{n\to\infty}\varphi(u_n^*) = -\infty$.

In the rest of this article, we first give some preliminaries in Section 2, then give the proof of Theorem 1.1 in Section 3.

2 Preliminaries

In this section, we first introduce some notations. Let **R, Z, N** be the sets of real numbers, integers and natural numbers, respectively. For $a, b \in \mathbf{Z}$ and $c \in \mathbf{R}$, $\mathbf{Z}[a, b]$ denotes the discrete interval $\{a, a + 1,..., b\}$ when $a < b$ and $[c]$ denote the largest integer less than c. In order to apply critical point theory, we then introduced the variational structure corresponding to system (1). For any given positive integer T, the linear space H_T is defined by

$$H_T = \{u : \mathbf{Z} \to \mathbf{R}^N | u(t) = u(t+T) \quad \text{for all } t \in \mathbf{Z}\}.$$

H_T can be equipped with inner product

$$\langle u, v\rangle = \sum_{t=0}^{T} (u(t), v(t)),$$

and the corresponding norm reads as

$$\|u\| = \left(\sum_{t=0}^{T} |u(t)|^2\right)^{\frac{1}{2}}.$$

It is easy to see that H_T is a finite dimensional Hilbert space and is linear homeomorphic to $\mathbf{R}^{NT}$. Define another norm $\|\cdot\|_\infty$ by

$$\|u\|_\infty = \max_{t\in\mathbf{Z}[0,T]} u(t).$$

Since H_T is finite dimensional, this norm is equivalent with $\|\cdot\|$:

$$\frac{1}{\sqrt{T}} \|u\| \le \|u\|_\infty \le \|u\|.$$

Consider the functional defined on H_T

$$\varphi(u) = \frac{1}{2}\sum_{t=0}^{T}\left|\Delta u(t)\right|^2 - \sum_{t=0}^{T}F(t, u(t)), \quad \forall u \in H_T.$$

One can easily check that $u \in H_T$ is a critical point of ϕ if and only if u is a solution of (1).

The following lemma give a new decomposition of H_T according to spectra of operator Δ^2 with periodic boundary condition.

Lemma 2.1 [[8], Lemma 1] *As a subspace of* H_T, N_k *is is defined by*

$$N_k := \{u \in H_T| - \Delta^2 u(t-1) = \lambda_k u(t)\},$$

where $\lambda_k = 2 - 2\cos k\omega,\ \omega = \frac{2\pi}{2},\ k \in \mathbf{Z}\,[0,\ [\frac{T}{2}]]$. *Then we have:*

(*a*) $N_k \perp N_j$ *for any* $k \neq j$ *and* $j,\ k \in \mathbf{Z}\,[0,\ [\frac{T}{2}]]$.

(*b*) $H_T = \oplus_{k=0}^{[\frac{T}{2}]}$.

Set $V = N_0$ and $W = \oplus_{k=1}^{[\frac{T}{2}]}N_k$. Then it is easy to see $H_T = V \oplus W$ and

$$\sum_{t=0}^{T}\left|\Delta u(t)\right|^2 \geq \lambda_1 \|u\| \quad \text{for any } u \in W.$$

The element u of V is just the eigenvector corresponding to $\lambda_0 = 0$ which satisfy $u(t) = u(0)$ for $t \in \mathbf{Z}\,[0, T]$.

Now we introduce a minimax theorem which include many well know results. This theorem not only asserts the existence of a Palais Smale sequence, but also gives the location information of the Palais Smale sequence. This will play an important role in our proof of Theorem 1.1.

Proposition 2.1 [[13], Corollary 4.3] *Let* K *be a compact metric space,* $K_0 \subset K$ *a closed set,* X *a Banach space,* $\chi \in C(K_0, X)$ *and let us define the complete metric space* M *by*

$$M = \{g \in C(K, X)|g(s) = \chi(s), \quad \forall s \in K_0\}$$

with the usual distance d. Let $\phi \in C^{\mathbf{1}}(X, \mathbf{R})$ *and let us define*

$$c = \inf_{g\in M}\max_{s\in K}\varphi(g(s)), \quad c_1 = \max_{s\in\chi(K_0)}\varphi(s).$$

If $c > c_1$, *then for each sequence* $\{f_k\} \subset M$ *such that* $\max_K \phi(f_k) \to c$, *there exists a sequence* $\{v_k\} \subset X$ *such that*

$$\varphi(v_k) \to c, \quad dist(v_k, f_k(K)) \to 0, \quad \left\|\varphi'(v_k)\right\| \to 0$$

as $k \to \infty$.

3 Proof of Theorem 1.1

In this section we give the proof of Theorem 1.1. Before that, we need to establish some basic lemmas.

Lemma 3.1 *Suppose* (F_α) *holds, then* ϕ *is coercive in the subspace* W, *that is* $\phi(u) \to \infty$ *as* $\|u\| \to \infty$ *in* W.

Proof: For any $u \in W$, it follows from (F_α) that

$$\begin{aligned}\varphi(u) &= \frac{1}{2}\sum_{t=0}^{T}\left|\Delta u(t)\right|^2 - \sum_{t=0}^{T}F(t,u(t)) \\ &\geq \frac{1}{2}\lambda_1\|u\|^2 - \sum_{t=0}^{T}\left(f(t)\left|u(t)\right|^{\alpha+1} + g(t)\left|u(t)\right|\right) \\ &\geq \frac{1}{2}\lambda_1\|u\|^2 - \|u\|_\infty^{\alpha+1}\sum_{t=0}^{T}f(t) - \|u\|_\infty\sum_{t=0}^{T}g(t) \\ &\geq \frac{1}{2}\lambda_1\|u\|^2 - \|u\|^{\alpha+1}\sum_{t=0}^{T}f(t) - \|u\|\sum_{t=0}^{T}g(t).\end{aligned}$$

Since $\alpha < 1$, we have $\phi(u) \to \infty$ as $\|u\| \to \infty$ in W.

Lemma 3.2 *Suppose* (3) *holds. Then there exists a positive sequence* $\{a_n\}$ *such that*

$$\lim_{n\to\infty} a_n = +\infty \text{ and } \lim_{n\to\infty}\sup_{u\in V,\|u\|=a_n}\varphi(u) = -\infty.$$

Proof. For $u \in V$, we have $u(0) = u(1) = \ldots = u(T)$, $\|u\|^2 = T|u(0)|^2$ and

$$\varphi(u) = -\sum_{t=0}^{T}F(t,u(t)) = -\sum_{t=0}^{T}F(t,u(0)).$$

By (3) there exists a sequence $\{d_n\}$ such that

$$\lim_{n\to\infty}\inf_{x\in\mathbf{R}^N,|x|=d_n}\sum_{t=0}^{T}F(t,x) = +\infty.$$

So if we choose $a_n = \sqrt{T}d_n$, then we have

$$\sup_{u\in V,\|u\|=a_n}\varphi(u) = \sup_{u\in V,\|u\|=a_n} -\sum_{t=0}^{T}F(t,u(0)) = -\inf_{u(0)\in\mathbf{R}^N,|u(0)|=d_n}\sum_{t=0}^{T}F(t,u(0)) \to -\infty$$

as $n \to \infty$.

Lemma 3.3 *Suppose* (F_α) *and* (4) *hold. Then there exists a positive sequence* $\{b_m\}$ *such that*

$$\lim_{m\to\infty} b_m = +\infty \text{ and } \lim_{m\to\infty}\inf_{u\in H_{b_m}}\varphi(u) = +\infty,$$

where $H_{b_m} = \{u \in V|\ \|u\| = b_m\} \oplus W$.

Proof. For any $u \in H_{b_m}$, let $u = \bar{u} + \tilde{u}$ with $\bar{u} \in V$ and $\tilde{u} \in W$. It follows form (F_α) that

$$\begin{aligned}
&\left|\sum_{t=0}^{T} F(t,u(t)) - \sum_{t=0}^{T} F(t,\bar{u}(t))\right| = \left|\sum_{t=0}^{T} \int_0^1 (\nabla F(t,\bar{u}(0)+s\tilde{u}(t)),\tilde{u}(t))\right| \\
&\leq \sum_{t=0}^{T} \left(f(t)\left|\bar{u}(0)+\tilde{u}(t)\right|^{\alpha} + g(t)\right)\left|\tilde{u}(t)\right| \\
&\leq 2\sum_{t=0}^{T} f(t)(\bar{u}(0)^{\alpha} + \tilde{u}(t)^{\alpha}\left|\tilde{u}(t)\right|) + \sum_{t=0}^{T} g(t)\left|\tilde{u}(t)\right| \\
&\leq 2(\bar{u}(0)^{\alpha} + \|\tilde{u}\|_{\infty}^{\alpha}))\|\tilde{u}\|_{\infty}\sum_{t=0}^{T} f(t) + \|\tilde{u}\|_{\infty}\sum_{t=0}^{T} g(t) \\
&\leq 2\left(\frac{\lambda_1}{8}\|\tilde{u}\|^2 + \frac{8}{\lambda_1}\|\bar{u}\|^{2\alpha}\right) + \|\tilde{u}\|^{1+\alpha}\sum_{t=0}^{T} f(t) + \|\tilde{u}\|\sum_{t=0}^{T} g(t) \\
&\leq \frac{\lambda_1}{4}\|\tilde{u}\|^2 + C\|\tilde{u}\|^{1+\alpha} + C\|\tilde{u}\| + C\|\bar{u}\|^{2\alpha}.
\end{aligned}$$

Substitute the above inequality into $\phi(u)$, we have

$$\begin{aligned}
\varphi(u) &= \frac{1}{2}\sum_{t=0}^{T}\left|\Delta u(t)\right|^2 - \sum_{t=0}^{T} F(t,u(t)) \\
&= \frac{1}{2}\sum_{t=0}^{T}\left|\Delta \tilde{u}(t)\right|^2 - \left(\sum_{t=0}^{T} F(t,u(t)) - \sum_{t=0}^{T} F(t,\bar{u}(t))\right) + \sum_{t=0}^{T} F(t,\bar{u}(t) \\
&\geq \frac{\lambda_1}{4}\|\tilde{u}\|^2 - C\|\tilde{u}\|^{1+\alpha} - C\|\tilde{u}\| + \|\bar{u}\|^{2\alpha}\left(\frac{\sum_{t=0}^{T} F(t,\bar{u}(t))}{\|\bar{u}\|^{2\alpha}} + C\right).
\end{aligned}$$

The sum of the first three terms is bounded form below. On the other hand, it follows form $(F_{\text{-}})$ there exists sequence $e_m \to \infty$ such that

$$\lim_{m\to\infty} \sup_{x\in\mathbf{R}^N, |x|=e_m} |x|^{-2\alpha}\sum_{t=0}^{T} F(t,x) = -\infty.$$

Hence, if we choose $b_m = \sqrt{T}e_m$, we have

$$\lim_{m\to\infty} \inf_{u\in H_{b_m}} \varphi(u) = +\infty.$$

After the above preparations we give the proof of our main result.

Proof of Theorem 1.1 Denote the ball in V with radius a_n by B_{a_n}. Then we define a family of maps

$$\Gamma_n = \{\gamma \in C(B_{a_n}, H) | \gamma|_{B_{a_n}} = Id|_{B_{a_n}}\}$$

and corresponding minimax values

$$c_n = \inf_{\gamma\in\Gamma_n} \max_{u\in B_{a_n}} \varphi(\gamma(u))$$

for each n. By Lemma 3.1, the functional ϕ is coercive on W, then there exists a constant M such that $\inf_{u\in W} \phi(u) \geq M$. On the other hand, it is well know that B_{a_n} and W

are linked (see [[14], Theorem 4.6]), i.e., for any $\gamma \in \Gamma_n$, $\gamma(B_{a_n}) \cap W \neq \emptyset$. It follows that $\max_{u\in B_{a_n}} \varphi(\gamma(u)) \geq \inf_{u\in W} \varphi(u)$ for any $\gamma \in \Gamma_n$. Hence we have $c_n \geq \inf_{u\in W} \phi(u) \geq M$. In view of Lemma 3.2,

$$c_n > \max_{u\in \partial B_{a_n}} \varphi(u)$$

holds for large values of n, where ∂B_{a_n} denote the boundary of B_{a_n} in $V: \{u \in V | \|u\| = a_n\}$.

For such n, there exists a sequence $\{\gamma_k\} \subset \Gamma_n$ such that

$$\max_{u\in B_{a_n}} \varphi(\gamma_k(u)) \to c_n \quad \text{as } k \to \infty.$$

Applying Proposition 2.1 with $X = H, K = B_{a_n}, K_0 = \partial B_{a_n}, \chi = Id$, we know there exists a sequence $\{v_k\} \subset H$ such that

$$\varphi(v_k) \to c_n, \quad \operatorname{dist}(v_k, \gamma_k(B_{a_n})) \to 0, \quad \left\|\varphi'(v_k)\right\| \to 0 \tag{5}$$

as $k \to \infty$. If we can show $\{v_k\}$ is bounded, then from the fact that H is finite dimensional we know there is a subsequence, which is still be denoted by $\{v_k\}$ such that v_k converge to some point u_n. By the continuity of ϕ and ϕ', we know $\phi(u_n) = c_n$ and $\phi'(u_n) = 0$. That is, u_n is a critical point of ϕ.

Now, let us show the sequence $\{v_k\}$ is bounded. For large enough k, by (5), we have

$$c_n \leq \max_{u\in B_{a_n}} \varphi(\gamma_k(u)) \leq c_n + 1,$$

and we can find $w_k \in \gamma_k(B_{a_n})$ such that $\|v_k - w_k\| \leq 1$. By Lemma 3.3, we can find a large enough m such that $b_m > a_n$ and $\inf_{u\in H_{b_m}} \varphi(u) > c_n + 1$. This implies that $\gamma_k(B_{a_n})$ can not intersect the hyperplane H_{b_m} for each k. Let $w_k = \bar{w}_k + \tilde{w}_k \in V$ with $\bar{w}_k \in V$ and $\tilde{w}_k \in W$. Then $\|\bar{w}_k\| < b_m$ for each k. Besides, by (F_α), it is obvious that

$$\begin{aligned}
c_n + 1 \geq \varphi(w_k) &= \frac{1}{2}\sum_{t=0}^{T} \left|\Delta w_k(t)\right|^2 - \sum_{t=0}^{T} F(t, w_k(t)) \\
&\geq \frac{1}{2}\lambda_1 \left\|\tilde{w}_k\right\|^2 - \sum_{t=0}^{T} \left(f(t)\left|w_k(t)\right|^{\alpha+1} + g(t)\left|w_k(t)\right|\right) \\
&\geq \frac{1}{2}\lambda_1 \left\|\tilde{w}_k\right\|^2 - 4\sum_{t=0}^{T} f(t)\left(\left|\bar{w}_k(0)\right|^{\alpha+1} + \left|\tilde{w}_k(t)\right|^{\alpha+1}\right) - \sum_{t=0}^{T} g(t)\left(\left|\bar{w}_k(0)\right| + \left|\tilde{w}_k(t)\right|\right) \\
&\geq \frac{1}{2}\lambda_1 \left\|\tilde{w}_k\right\|^2 - 4\left\|\tilde{w}_k\right\|^{\alpha+1} \sum_{t=0}^{T} f(t) - \left\|\tilde{w}_k\right\| \sum_{t=0}^{T} g(t) - 4b_m^{\alpha+1} \sum_{t=0}^{T} f(t) - b_m \sum_{t=0}^{T} g(t).
\end{aligned} \tag{6}$$

This implies that $\left\|\tilde{w}_k\right\|$ is bounded too. From $\|w_k\| \leq C(\|\bar{w}_k\| + \left\|\tilde{w}_k\right\|)$ we know w_k is bounded. Hence $\{v_k\}$ is bounded. From previous discussion we know that the accumulation point u_n of $\{v_k\}$ is a critical point and c_n is critical value of ϕ. In order to prove part (a), we still have to show

$$\lim_{n\to\infty} c_n = +\infty. \tag{7}$$

Note that if we choose large enough n such that $a_n > b_m$, then $\gamma(B_{a_n})$ intersect the hyperplane H_{b_m} for any $\gamma \in \Gamma_n$. It follows that

$$\max_{u\in B_{a_n}} \varphi(\gamma(u)) \geq \inf_{u\in H_{b_m}} \varphi(u).$$

This inequality and Lemma 3.3 implies (7).

Next we prove part (*b*) of Theorem 1.1. For fixed $m \in \mathbf{N}$, define the subset P_m of H by

$$P_m = \{u = \bar{u} + \tilde{u} \in H|\bar{u} \in V, \quad \|\bar{u}\| \leq b_m, \quad \tilde{u} \in W\}.$$

It follows form (6) that ϕ is bounded form below on P_m. Let us set

$$\mu_m = \inf_{u\in P_m} \varphi(u)$$

and choose a minimizing sequence $\{u_k\}$ in P_m, that is,

$$\varphi(u_k) \to \mu_m \quad \text{as } m \to \infty.$$

From (6) we know that $\{u_k\}$ is bounded in H. Then there exists a subsequence, which is still be denoted by $\{u_k\}$ such that $u_k \to u_m^*$ as $k \to \infty$. Form the fact that P_m is a closed subset of H and that ϕ is continuous we know $u_m^* \in P_m$ and

$$\mu_m = \lim_{k\to\infty} \varphi(u_k) = \varphi(u_m^*).$$

If we can show u_m^* is in the interior of P_m, then u_m^* is a critical point of ϕ. Let $u_m^* = \bar{u}_m^* + \tilde{u}_m^*$ with $\bar{u}_m^* \in V$ and $\tilde{u}_m^* \in W$. If $a_n < b_m$, then $\partial B_{a_n} \subset P_m$. This implies that

$$\varphi(u_m^*) = \inf_{u\in P_m} \varphi(u) \leq \sup_{u\in\partial B_{a_n}} \varphi(u).$$

It follows from Lemma 3.2 that $\varphi(u_m^*) \to -\infty$ as $m \to \infty$. By Lemma 3.3 we have $\bar{u}_m^* \neq b_m$ for large values of m, which means that u_m^* is in the interior of P_m, and u_m^* is a critical point of ϕ with $\varphi(u_m^*) \to -\infty$ as $m \to \infty$. The proof of Theorem 1.1 is finished.

Example 1 *Now we give an example of potential function which satisfies condition* (F_α), (3) *and* (4). *For simplicity, we drop the dependence in t:*

$$F(x) = |x|^{1+\alpha} \sin(\log(1 + |x|)).$$

Note that F is continuously differentiable and its gradient reads as

$$\nabla F(x) = |x|^{\alpha-1} \sin(\log(1 + |x|))x + \frac{|x|^\alpha \cos(\log(1 + |x|))}{1 + |x|} x.$$

Then it is easy to see F satisfies condition (F_α), (3) *and* (4). *A routine application of Theorem* 1.1 *shows that system* (1) *with potential function F has infinitely many periodic solutions.*

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Authors' contributions
CC chose the problem in this paper and drafted the manuscript. XX conceived the solution method. Both authors read and approved the final manuscript.

Competing interests
The authors declare that they have no competing interests.

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References
1. Pankov, A, Zakharchenko, N: On some discrete variational problems. Acta Appl Math. **65**, 295–303 (2000)
2. Pankov, A, Rothos, V: Solitons in discrete nonlinear Schrodinger equation with saturable non-linearity. Proc R Soc Lond Ser A. **464**, 3219–3236 (2008)
3. Guo, Z, Yu, J: Existence of periodic and subharmonic solutions for second-order superlinear difference equations. Sci China Ser A. **46**(4):506–515 (2003)
4. Guo, Z, Yu, J: Periodic and subharmonic solutions for superquadratic discrete Hamiltonian systems. Nonlinear Anal. **55**(7-8):969–983 (2003)
5. Guo, Z, Yu, J: The existence of periodic and subharmonic solutions of subquadratic second-order difference equations. J Lond Math Soc. **68**(2):419–430 (2003)
6. Xue, YF, Tang, CL: Existence of a periodic solution for subquadratic second-order discrete Hamiltonian system. Nonlinear Anal. **6**(7):2072–2080 (2007)
7. Guo, Z, Yu, J: Multiplicity results for periodic solutions to second-order difference equations. J Dyn Diff Equ. **18**(4):943–960 (2006)
8. Xue, YF, Tang, CL: Multiple periodic solutions for superquadratic second-order discrete Hamiltonian systems. Appl Math Comput. **196**, 494–500 (2008)
9. Zhou, Z, Guo, Z, Yu, J: Periodic solutions of higher-dimensional discrete systems. Proc R Soc Edinburgh. **134A**, 1013–1022 (2004)
10. Liu, J: A geometrical index for the group Z_p. Acta Math Sinica, New Ser. **5**(3):193–196 (1989)
11. Habets, P, Manásevich, R, Zanolin, F: A nonlinear boundary value problem with potential oscillating aroud the first eigenvalue. J Diff Equ. **117**, 428–445 (1995)
12. Zhang, P, Tang, CL: Infinitely many periodic solutions for nonautonomous sublinear second order Hamiltonian systems. Abstr Appl Anal. **2010**, 1–10 (2010)
13. Mawhin, J, Willem, M: Critical Point Theory and Hamiltonian System. Springer-Verlag, New York (1989)
14. Rabinowitz, P: Minimax Methods in Critical Point Theory with Application to differential Equations. AMS, Providence. (1986)

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