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Some results on q-difference equations

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Abstract

In this paper, we consider the q-difference analogue of the Clunie theorem. We obtain there is no zero-order entire solution of $f^n(z) + (\nabla_q f(z))^n = 1$ when $n \geq 2$; there is no zero-order transcendental entire solution of $f^n(z) + P(z)(\nabla_q f(z))^m = Q(z)$ when $n > m \geq 0$; and the equation $f^n + P(z)\nabla_q f(z) = h(z)$ has at most one zero-order transcendental entire solution f if f is not the solution of $\nabla_q f(z) = 0$, when $n \geq 4$.

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1 Introduction and main results

It is well known that Clunie's theorem (see [1], Lemma 1; also see [2], p.39, Lemma 2.4.1) is a useful tool in studying complex differential equations. It states that $Q_n(f)$ is a polynomial of total degree n at most in the meromorphic function f and its derivatives having meromorphic functions as coefficients. If $T(r)$ is the maximum of the characteristics of the coefficients, then

$$\frac{1}{2\pi}\int_{|f|>1} \log^+ \bigl|f^{-n}Q_n(f)\bigr|\, d\varphi = O\bigl(\log r + \log T(r,f) + T(r)\bigr) \quad \text{n.e. as } r \to \infty.$$

Later, Clunie's theorem has been improved into many forms (see [2], pp.39-44) which are valuable tools for studying meromorphic solutions of Painlevé and other non-linear differential equations; see, *e.g.*, [2].

In 2007, Laine and Yang [3] obtained a discrete version of Clunie's theorem.

Theorem A *Let f be a transcendental meromorphic solution of finite-order ρ of a difference equation of the form*

$$U(z,f)P(z,f) = Q(z,f),$$

where $U(z,f)$, $P(z,f)$, and $Q(z,f)$ are difference polynomials such that the total degree $\deg U(z,f) = n$ in $f(z)$ and its shifts, and $\deg Q(z,f) \leq n$. Moreover, we assume that $U(z,f)$ contains just one term of maximal total degree in $f(z)$ and its shifts. Then for each $\varepsilon > 0$,

$$m\bigl(r, P(z,f)\bigr) = O\bigl(r^{\rho-1+\varepsilon}\bigr) + S(r,f)$$

possibly outside of an exceptional set of finite logarithmic measure.

Now let us introduce some notation. Let $c_j \in \mathbb{C}$ for $j = 1, \dots, n$, and let I be a finite set of multi-indexes $\lambda = (\lambda_0, \dots, \lambda_n)$. A difference polynomial of a meromorphic function $w(z)$ is defined as

$$\begin{aligned} P(z,w) &= P\bigl(z, w(z), w(z+c_1), \dots, w(z+c_n)\bigr) \\ &= \sum_{\lambda\in I} a_\lambda(z) w(z)^{\lambda_0} w(z+c_1)^{\lambda_1} \cdots w(z+c_n)^{\lambda_n}, \end{aligned} \tag{1.1}$$

where the coefficients $a_\lambda(z)$ are small with respect to $w(z)$ in the sense that $T(r, a_\lambda) = o(T(r,w))$ as r tends to infinity outside of an exceptional set E of finite logarithmic measure

$$\lim_{r\to\infty} \int_{E\cap[1,r)} \frac{dt}{t} < \infty.$$

The total degree of $P(z,w)$ in $w(z)$ and in the shifts of $w(z)$ is denoted by $\deg_w(P)$, and the order of a zero of $P(z, x_0, x_1, \dots, x_n)$, as a function of x_0 at $x_0 = 0$, is denoted by $\operatorname{ord}_0(P)$; see, *e.g.*, [4]. Moreover, the weight of a difference polynomial (1.1) is defined by

$$K(P) = \max_{\lambda\in I} \left\{ \sum_{j=1}^{n} \lambda_j \right\},$$

where λ and I are the same as in (1.1) above. The difference polynomial $P(z,w)$ is said to be homogeneous with respect to $w(z)$ if the degree $d_\lambda = \lambda_0 + \cdots + \lambda_n$ of each term in the sum (1.1) is non-zero and the same for all $\lambda \in I$.

Recently, Korhonen obtained a new Clunie-type theorem in [4].

Theorem B *Let $w(z)$ be a finite-order meromorphic solution of*

$$H(z,w)P(z,w) = Q(z,w),$$

where $P(z,w)$ is a homogeneous difference polynomial with meromorphic coefficients, and $H(z,w)$ and $Q(z,w)$ are polynomials in $w(z)$ with meromorphic coefficients having no common factors. If

$$\max\bigl\{\deg_w(H), \deg_w(Q) - \deg_w(P)\bigr\} > \min\bigl\{\deg_w(P), \operatorname{ord}_0(Q)\bigr\} - \operatorname{ord}_0(P),$$

then $N(r,w) \neq S(r,w)$.

Theorem C *Let $w(z)$ be a finite-order meromorphic solution of*

$$H(z,w)P(z,w) = Q(z,w),$$

where $P(z,w)$ is a homogeneous difference polynomial with meromorphic coefficients, and $H(z,w)$ and $Q(z,w)$ are polynomials in $w(z)$ with meromorphic coefficients having no common factors. If

$$2K(P) \le \max\bigl\{\deg_w(Q), \deg_w(H) + \deg_w(P)\bigr\} - \min\bigl\{\deg_w(P), \operatorname{ord}_0(Q)\bigr\},$$

then for any $\delta \in (0,1)$,

$$m(r,w) = o\left(\frac{T(r,w)}{r^{\delta}}\right) + O\big(T(r)\big),$$

where r goes to infinity outside of an exceptional set of finite logarithmic measure, and $T(r)$ *is the maximum of the Nevanlinna characteristics of the coefficients of* $P(z,w)$, $Q(z,w)$, *and* $H(z,w)$.

The non-autonomous Schröder q-difference equation

$$f(qz) = R\big(z,f(z)\big), \tag{1.2}$$

where the right-hand side is rational in both arguments, has been widely studied during the last decades; see, *e.g.*, [5–8]. Gundersen *et al.* [9] considered the order of growth of meromorphic solutions of (1.2), from which a q-difference analogue of the classical Malmquist theorem [10] is given: if the q-difference equation (1.2) admits a meromorphic solution of order zero, then (1.2) reduces to a q-difference Riccati equation, *i.e.*, $\deg_f R = 1$.

Bergweiler *et al.* [11] treated the functional equation

$$\sum_{j=0}^{n} a_j(z)f\big(c^j z\big) = Q(z), \tag{1.3}$$

where $0 < |c| < 1$ is a complex number, $a_j(z)$ ($j = 0,1,\ldots,n$), and $Q(z)$ are rational functions with $a_0(z) \not\equiv 0$, $a_1(z) \equiv 1$. They concluded that all meromorphic solutions of (1.3) satisfy $T(r,f) = O((\log r)^2)$. This implies that all meromorphic solutions of (1.3) are of zero order of growth.

Let us recall $\Delta_c f(z)$ of a meromorphic function in the whole plane $\mathbb{C}$ is given by

$$\Delta_c f(z) = f(z+c) - f(z),$$

while

$$\nabla_q f(z) = f(qz) - f(z)$$

denotes $\nabla_q f(z)$ of a meromorphic function in the whole plane $\mathbb{C}$, where $c \in \mathbb{C}\backslash\{0\}$ and $q \in \mathbb{C}\backslash\{0,1\}$. The upper logarithmic density of E is defined by

$$\overline{\log \operatorname{dens}(E)} = \limsup_{r\to\infty} \frac{\int_{E\cup[1,r]} \frac{dt}{t}}{\log r}.$$

In particular, we denote by $S_q(r,f)$ any quantity satisfying $S_q(r,f) = o(T(r,f))$ for all r outside of a set of upper logarithmic density 0 on the set of logarithmic density 1.

In 2009, Liu [12] proved the following the result.

Theorem D *There is no non-constant entire solution with finite order of the non-linear difference equation*

$$f^2(z) + \big(\Delta_c f(z)\big)^2 = 1. \tag{1.4}$$

It is well known that $f^n(z) + g^n(z) = 1$ has no entire solutions when $n \geq 3$ (see [13], Theorem 3), and from Theorem D, we can say there is no non-constant entire solutions with finite order of the equation $f^n(z) + (\Delta_c f(z))^n = 1$, when $n \geq 2$.

In this paper, we replace $\Delta_c f(z)$ by $\nabla_q f(z)$ and get the following result.

Theorem 1 *There is no non-constant entire solution with zero order of the non-linear q-difference equation*

$$f^n(z) + \left(\nabla_q f(z)\right)^n = 1, \tag{1.5}$$

when $n \geq 2$.

Theorem 2 *Let* $P(z)$ *and* $Q(z)$ *be polynomials, and let* n *and* m *be integers satisfying* $n > m \geq 0$. *Then there is no non-constant entire transcendental solution with zero order of the non-linear q-difference equation*

$$f^n(z) + P(z)\left(\nabla_q f(z)\right)^m = Q(z). \tag{1.6}$$

In 2010, Yang and Laine [14] got the following result.

Theorem E *Let* $n \geq 4$ *be an integer,* $M(z,f)$ *be a linear difference polynomial of* f, *not vanishing identically, and* h *be a meromorphic function of finite order. Then the difference equation*

$$f^n + M(z,f) = h$$

possesses at most one admissible transcendental entire solution of finite order such that all coefficients of $M(z,f)$ *are small functions of* f. *If such a solution* f *exists, then* f *is of the same order as* h.

In this paper, we replace difference polynomial by q-difference polynomial and get the following result.

Theorem 3 *Let* $n \geq 4$ *be an integer,* $\widetilde{M}(r,f)$ *be a linear q-difference polynomial of* f, *not vanishing identically, and* $h(z)$ *be a meromorphic function. Suppose* $f(z)$ *is the solution of q-difference equation*

$$f^n + N(r,f) = h(z). \tag{1.7}$$

If $f(z)$ *is not the solution of* $\widetilde{M}(r,f) = 0$, *then equation* (1.7) *possesses at most one transcendental entire solution of zero order.*

2 Auxiliary results

The following auxiliary results will be instrumental in proving the theorems.

Lemma 1 ([5], Theorem 1.2) *Let $f(z)$ be a non-constant zero-order meromorphic function and $q \in \mathbb{C}\setminus\{0\}$. Then*

$$m\left(r, \frac{f(qz)}{f(z)}\right) = S_q(r,f).$$

Lemma 2 ([5], Theorem 2.1) *Let $f(z)$ be a non-constant zero-order meromorphic solution of*

$$f^n(z)P(z,f) = Q(z,f),$$

where $P(z,f)$ and $Q(z,f)$ are q-difference polynomials in $f(z)$. If the degree of $Q(z,f)$ as a polynomial in $f(z)$ and its q-shifts is at most n, then

$$m\big(r, P(z,f)\big) = S_q(r,f).$$

Lemma 3 ([15], Theorem 1.1) *Let $f(z)$ be a non-constant zero-order meromorphic function and $q \in \mathbb{C}\setminus\{0\}$. Then*

$$T\big(r, f(qz)\big) = T(r,f) + S_q(r,f).$$

Lemma 4 ([15], Theorem 1.3) *Let $f(z)$ be a non-constant zero-order meromorphic function and $q \in \mathbb{C}\setminus\{0\}$. Then*

$$N\big(r, f(qz)\big) = N(r,f) + S_q(r,f).$$

Lemma 5 ([16], Lemma 4) *If $T : \mathbb{R}^+ \to \mathbb{R}^+$ is a piecewise continuous increasing function such that*

$$\lim_{r\to\infty} \frac{\log T(r)}{\log r} = 0,$$

then the set

$$E := \big\{r : T(C_1 r) \geq C_2 T(r)\big\}$$

has logarithmic density 0 for all $C_1 > 1$ and $C_2 > 1$.

Lemma 6 *Let $f(z)$ be a zero-order entire function, $q \in \mathbb{C}\setminus\{0\}$, and a be a non-zero constant. If $f(z)$ and $\nabla_q f(z)$ share the set $\{a, -a\}$ CM, then $f(z)$ is a constant.*

Proof Since $f(z)$ is an entire function of zero order, and $f(z)$ and $\nabla_q f(z)$ share the set $\{a, -a\}$ CM, it is immediate to conclude that

$$\frac{(f(z) - a)(f(z) + a)}{(\nabla_q f(z) + a)(\nabla_q f(z) - a)} \equiv k^2, \tag{2.1}$$

where k is a constant.

If $k^2 \neq 1$, let

$$h_1(z) := f(z) + k\nabla_q f(z), \tag{2.2}$$

and

$$h_2(z) := f(z) - k\nabla_q f(z). \tag{2.3}$$

Then $h_1(z)$ and $h_2(z)$ are entire functions, and

$$f(z) = \frac{h_1(z) + h_2(z)}{2}, \qquad \nabla_q f(z) = \frac{h_1(z) - h_2(z)}{2k}. \tag{2.4}$$

From (2.1)-(2.3), we have

$$h_1(z)h_2(z) = f^2(z) - k^2\bigl(\nabla_q f(z)\bigr)^2 = \bigl(1 - k^2\bigr)a^2. \tag{2.5}$$

From (2.2), (2.3), and (2.5), we obtain

$$N\left(r, \frac{1}{h_1}\right) = N\left(r, \frac{1}{h_2}\right) = 0. \tag{2.6}$$

Since h_1 and h_2 with zero order have no zeros and no poles, both h_1 and h_2 are constants. (2.4) implies that $f(z)$ is a constant.

If $k^2 = 1$, from (2.1) we get $f(z) = \nabla_q f(z)$. According to Lemma 3, it implies that $f(z)$ must be a constant. □

3 Clunie theorem for q-difference

Let us consider the q-difference polynomial case. Let $d_j \in \mathbb{C}$ for $j = 1, \dots, n$, and let I_q be a finite set of multi-indexes $\gamma = (\gamma_0, \dots, \gamma_n)$. A difference polynomial of a meromorphic function $w(z)$ is defined as

$$\begin{aligned} P(z,w) &= P\bigl(z, w(qz), w\bigl(q^2 z\bigr), \dots, w\bigl(q^n z\bigr)\bigr) \\ &= \sum_{\gamma \in I_q} a_\gamma(z) w(z)^{\gamma_0} w(qz)^{\gamma_1} \dots w\bigl(q^n z\bigr)^{\gamma_n}, \end{aligned} \tag{3.1}$$

where the coefficients $a_\gamma(z)$ are small with respect to $w(z)$ in the sense that $T(r, a_\gamma) = o(T(r,w))$ as r tends to infinity outside of an exceptional set E of finite logarithmic measure

$$\lim_{r\to\infty} \int_{E\cap[1,r)} \frac{dt}{t} < \infty.$$

The total degree of $P(z,w)$ in $w(z)$ and in the q-shifts of $w(z)$ is denoted by $\deg_w^q(P)$, and the order of a zero of $P(z, x_0, x_1, \dots, x_n)$, as a function of x_0 at $x_0 = 0$, is denoted by $\operatorname{ord}_0^q(P)$; see, *e.g.*, [4]. Moreover, the weight of a difference polynomial (1.1) is defined by

$$K_q(P) = \max_{\gamma \in I_q} \left\{ \sum_{j=1}^{n} \gamma_j \right\},$$

where γ and I_q are the same as in (3.1) above. The difference polynomial $P(z,w)$ is said to be homogeneous with respect to $w(z)$, if the degree $d_\gamma = \gamma_0 + \cdots + \gamma_n$ of each term in the sum (1.1) is non-zero and the same for all $\gamma \in I_q$.

In this paper, we will obtain the new Clunie theorem for q-difference polynomials.

Theorem 4 *Let $w(z)$ be a zero-order meromorphic solution of*

$$H(z,w)P(z,w) = Q(z,w),$$

where $P(z,w)$ is a homogeneous q-difference polynomial with polynomial coefficients, and $H(z,w)$ and $Q(z,w)$ are polynomials in $w(z)$ with polynomial coefficients having no common factors. If

$$\max\left\{\deg_w^q(H), \deg_w^q(Q) - \deg_w^q(P)\right\} > \min\left\{\deg_w^q(P), \operatorname{ord}_0^q(Q)\right\} - \operatorname{ord}_0^q(P),$$

then $N(r,w) \neq S_q(r,w)$.

Proof Since $P(r,w)$ is homogeneous, by Lemma 1 it follows that

$$m\left(r, \frac{P(r,w)}{w^{\deg_w^q(P)}}\right) = S_q(r,w). \tag{3.2}$$

Moreover, Mohon'ko's theorem (see [17], Theorem 1.13) implies that

$$T\left(r, \frac{P(r,w)}{w^{\deg_w^q(P)}}\right) = d_w T(r,w) + O(\log r), \tag{3.3}$$

where

$$d_m = \max\left\{\deg_w^q(Q), \deg_w^q(H) + \deg_w^q(P)\right\} - \min\left\{\deg_w^q(P), \operatorname{ord}_0^q(Q)\right\}. \tag{3.4}$$

According to (3.2), (3.3), (3.4), and the assumption of Theorem 4, it follows that

$$N\left(r, \frac{P(r,w)}{w^{\deg_w^q(P)}}\right) \geq \left(1 + \deg_w^q(P) - \operatorname{ord}_0^q(P)\right) T(r,w) + S_q(r,w). \tag{3.5}$$

This contradicts the assertion of Theorem 4 that $N(r,w) = S_q(r,w)$. Let us denote $q_{\max} = \max\{|q_j|, (j = 1, \ldots, n)\}$, by Lemma 5 we will obtain that

$$\begin{aligned} N\left(r, \frac{P(r,w)}{w^{\operatorname{ord}_0^q(P)}}\right) &\leq \left(\deg_w^q(P) - \operatorname{ord}_0^q(P)\right) N(q_{\max} r, w) \\ &\leq \left(\deg_w^q(P) - \operatorname{ord}_0^q(P) + o(1)\right) N(r,w) + S_q(r,w) \\ &= S_q(r,w). \end{aligned}$$

Therefore,

$$\begin{aligned} N\left(r, \frac{P(r,w)}{w^{\deg_w^q(P)}}\right) &\leq N\left(r, \frac{P(r,w)}{w^{\operatorname{ord}_0^q(P)}}\right) + N\left(r, \frac{1}{w^{\deg_w^q(P) - \operatorname{ord}_0^q(P)}}\right) \\ &= N\left(r, \frac{1}{w^{\deg_w^q(P) - \operatorname{ord}_0^q(P)}}\right) + S_q(r,w) \end{aligned}$$

$$
\begin{aligned}
&\le T\left(r, \frac{1}{w^{\deg_w^q(P)-\operatorname{ord}_0^q(P)}}\right) + S_q(r,w) \\
&= \left(\deg_w^q(P) - \operatorname{ord}_0^q(P)\right) T(r,w) + S_q(r,w),
\end{aligned}
$$

which is a contradiction to (3.5). We can conclude that $N(r,w) \neq S_q(r,w)$. □

Theorem 5 *Let $w(z)$ be a zero-order meromorphic solution of*

$$H(z,w)P(z,w) = Q(z,w),$$

where $P(z,w)$ is a homogeneous q-difference polynomial with polynomial coefficients, and $H(z,w)$ and $Q(z,w)$ are polynomials in $w(z)$ with polynomial coefficients having no common factors. If

$$2K_q(P) \le \max\left\{\deg_w^q(Q), \deg_w^q(H) + \deg_w^q(P)\right\} - \min\left\{\deg_w^q(P), \operatorname{ord}_0^q(Q)\right\},$$

then

$$m(r,w) = S_q(r,w).$$

Proof On the one hand, (3.2) and (3.5) imply that

$$N\left(r, \frac{P(r,w)}{w^{\deg_w^q(P)}}\right) = d_w T(r,w) + S_q(r,w). \tag{3.6}$$

On the other hand, by Lemma 4, we can obtain that

$$N\left(r, \frac{P(r,w)}{w^{\deg_w^q(P)}}\right) \le K_q(P)\left(2T(r,w) - m(r,w)\right) + S_q(r,w). \tag{3.7}$$

(3.6) and (3.7) show that

$$m(r,w) = S_q(r,w).$$ □

4 Proof of Theorem 1

If $n \ge 3$, there are no non-constant entire solutions of equation (1.5) according to [13]. Let us consider $n = 2$, that is,

$$f^2(z) + \left(\Delta_q f(z)\right)^2 = 1. \tag{4.1}$$

From (4.1), we get $f(z)$ and $\nabla_q f(z)$ share the set $\{\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\}$ CM. From Lemma 6, we obtain that $f(z)$ is a constant.

5 Proof of Theorem 2

Suppose that f is a transcendental entire solution of equation (1.6) with zero order. If $\nabla_q f(z) \equiv 0$, then $f^n(z) = Q(z)$, and the conclusion holds. If $\nabla_q f(z) \not\equiv 0$, we have

$$f^{n-1} f = Q(z) - P(z)\frac{(\Delta_q f(z))}{f^m} f^m.$$

From Lemma 1, Lemma 2, and the condition $n > m$, we have

$$T(r,f) = m(r,f) = S_q(r,f),$$

which is impossible.

6 Proof of Theorem 3

Assume now, contrary to the assertion, that f and g, which are not the solutions of $\widetilde{M}(r,f) = 0$ and $\widetilde{M}(r,g) = 0$, are two distinct zero-order transcendental entire solutions of (1.7), then we can write

$$f^n + \widetilde{M}(r,f) = g^n + \widetilde{M}(r,g). \tag{6.1}$$

From (6.1), we obtain

$$f^n - g^n = \widetilde{M}(r,g) - \widetilde{M}(r,f). \tag{6.2}$$

Therefore, we have

$$F := \frac{f^n(z) - g^n(z)}{f(z) - g(z)} = \prod_{j=1}^{n-1}(f - \eta_j g) = -\frac{\widetilde{M}(r,f) - \widetilde{M}(r,g)}{f(z) - g(z)} \tag{6.3}$$

is an entire function, and $\eta_1, \ldots, \eta_{n-1}$ are distinct roots $\neq 1$ of the equation $z^n = 1$. Hence, $N(r, \frac{\widetilde{M}(r,f)-\widetilde{M}(r,g)}{f(z)-g(z)}) = 0$. From Lemma 1, we get

$$T\left(r, \frac{\widetilde{M}(r,f) - \widetilde{M}(r,g)}{f(z) - g(z)}\right) = S_q(r,f-g). \tag{6.4}$$

If $\frac{f}{g}$ is not a constant, (6.3) implies that

$$(n-1)T\left(r,\frac{f}{g}\right) = T\left(r,g^{n-1}\right) + S_q(r,f-g). \tag{6.5}$$

Thus,

$$\begin{aligned}\sum_{j=1}^{n-1} N\left(r, \frac{1}{\frac{f}{g} - \eta_j}\right) &= N\left(r, \frac{f-g}{\widetilde{M}(r,f) - \widetilde{M}(r,g)}\right) \\ &\leq T\left(r, \frac{f-g}{\widetilde{M}(r,f) - \widetilde{M}(r,g)}\right) \\ &= S_q(r,f-g).\end{aligned}$$

From the second fundamental theorem, we have

$$(n-3)T\left(r,\frac{f}{g}\right) \leq \sum_{j=1}^{n-1} N\left(r, \frac{1}{\frac{f}{g} - \eta_j}\right) + O(\log r) = S_q(r,f-g). \tag{6.6}$$

From (6.6), (6.5), and f, g are zero-order entire functions, we get a contradiction. Therefore, $\frac{f}{g}$ must be a constant. If $f/g = k \neq \eta_j$, k is a constant, then from (6.1) we have

$$\left(k^n - 1\right)g^n = \widetilde{M}\left(r, (1-k)g\right). \tag{6.7}$$

From Lemma 2 and (6.7), we get a contradiction. Thus, $f/g = \eta_j$ for some $j = 1, \ldots, n-1$ and

$$f(z) = \eta_j g(z). \tag{6.8}$$

This implies that

$$f^n = g^n \tag{6.9}$$

and

$$\widetilde{M}(r,f) = \eta_j \widetilde{M}(r,g). \tag{6.10}$$

From (6.9) and (6.2), we obtain

$$\widetilde{M}(r,f) = \widetilde{M}(r,g). \tag{6.11}$$

From (6.9) and (6.2), it is easy to get $\widetilde{M}(r,f) = 0$ and $\widetilde{M}(r,g) = 0$, which is impossible.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
The first author discovered some essential ideas for the proof of this paper, and made the actual writing. The fours discussed the paper together. The other authors checked the proofs of the paper. All authors read and approved the final manuscript.

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