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Some improved criteria on exponential stability of neutral differential equation

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Abstract

In this paper, some improved delay-dependent exponential stability criteria of the neutral delayed differential equation

$$\frac{d}{dt}\big[x(t) + px(t - \tau(t))\big] + ax(t) - b\tanh x(t - \sigma(t)) = 0,$$

are expressed in terms of linear matrix inequalities (LMIs). Compared with some existing ones, our results are derived without the use of the mode transformation method and the bounding technique. Three numerical examples are provided to show the effectiveness of the proposed method and less conservatism of the obtained results.

Keywords: exponential stability; neutral differential equations; linear matrix inequalities (LMIs); time-varying delays

1 Introduction

In this paper, we reconsider the following neutral delayed differential equation:

$$\frac{d}{dt}\big[x(t) + px(t - \tau(t))\big] + ax(t) - b\tanh x(t - \sigma(t)) = 0, \tag{1}$$

for $t \geq 0$, where a, b are two positive real constants and $|p| < 1$. The delays $\tau(\cdot) : [0, +\infty) \to [0, \tau]$ ($\tau > 0$) and $\sigma(\cdot) : [0, +\infty) \to [0, \sigma]$ ($\sigma > 0$) are bounded functions and $r = \max\{\tau, \sigma\} > 0$. There exist two positive constants $\mu_1, \mu_2 \in (0, 1)$ such that $\tau'(t) \leq \mu_1$, $\sigma'(t) \leq \mu_2$. For each solution of equation (1), we assume the initial condition

$$x_0(\theta) = \phi(\theta), \quad \theta \in [-r, 0],$$

where $\phi \in C([-r, 0]; R)$.

When $\tau(t) \equiv \tau$ and $\sigma(t) \equiv \sigma$, the system (1) is turned into the following form:

$$\frac{d}{dt}\big[x(t) + px(t - \tau)\big] + ax(t) - b\tanh x(t - \sigma) = 0, \quad t \geq 0. \tag{2}$$

Recently, for equation (2), many authors have investigated its properties due to its applications in the research of the dynamic characteristics of neural networks of Hopfield type,

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see [1–17] and references therein. Although some qualitative stability analysis for equation (2) have been presented in [1, 3, 5–14, 17], their results are mainly concerned with the asymptotic stability, without providing any conditions for exponential stability and any information about the decay rates of equation (2). Besides, it should be pointed out that the advantage of equation (2) with exponential stability in comparison with that with asymptotic stability lies in that the former can provide fast convergence and desirable accuracy. In [8], Li has obtained the global exponential stability for equation (2), but the sufficient condition is delay-independent. It is well known that current efforts on the problem of stability for delay systems can be divided into two categories, namely delay-independent stability criteria and delay-dependent stability criteria. And generally speaking, the former are more conservative than the latter when the delay is small. Thus, the sufficient condition about the global exponential stability for equation (2) given in [8] is more conservative. More recently, in [16], Rojsiraphisal and Niamsup have proposed some delay-dependent LMI-based sufficient conditions ensuring the exponential stability for equation (2), but these conditions are also more conservative since the mode transformation and the bounding technique are both used. On the other hand, to the best of our knowledge, compared with constant-delay systems, the time-varying delay systems are more closed to reflect the reality. So, it is necessary to discuss the global exponential stability for equation (1). In [18], Chen and Meng have established the LMI-based exponential stability criterion for equation (1) by only constructing a modified Lyapunov functional, but the result in [18] is sightly more conservative. Thus, there still exists room for further improvement.

In this paper, we reconsider the exponential stability of equation (1). New delay-dependent sufficient criteria ensuring the global exponential stability for equation (1) are given in terms of linear matrix inequalities (LMIs). The criteria here are also discussed from the point of view of its comparison with the earlier results. To show the applicability and effectiveness of the proposed method developed in this paper, three illustrative examples are provided.

Notation Throughout this paper, the notation $X \geq Y$ (respectively, $X > Y$) means that the matrix $X - Y$ is positive semi-definite (respectively, positive definite), where X and Y are symmetric matrices of the same dimensions. The symbol $*$ denotes the elements below the main diagonal of a symmetric matrix.

2 Main results

In this section, we give our main results.

Theorem 1 *For a given positive constant $\kappa > 0$, the zero solution of equation* (1) *is globally exponentially stable if there exist some positive scalars*: $\alpha_0, \alpha_1, \alpha_2, \alpha_3$ *and* α_4, *such that the*

following linear matrix inequalities (*LMIs*) *hold*:

$$
\Omega_1 = \begin{bmatrix} \Omega_{11}^1 & \Omega_{12}^1 & \lambda_1 \alpha_0 b & 0 \\ * & \Omega_{22}^1 & \lambda_1 \alpha_0 p b & 0 \\ * & * & -\lambda_1 \alpha_3 (1 - \mu_2) & 0 \\ * & * & * & -\alpha_2 \tau \end{bmatrix} < 0,
$$

$$
\Omega_2 = \begin{bmatrix} \Omega_{11}^2 & \Omega_{12}^2 & \lambda_2 \alpha_0 b & 0 \\ * & \Omega_{22}^2 & \lambda_2 \alpha_0 p b & 0 \\ * & * & -\alpha_3 \lambda_2 (1 - \mu_2) & 0 \\ * & * & * & -\alpha_4 \sigma \end{bmatrix} < 0,
$$

(3)

where

$$
\Omega_{11}^1 = 2\kappa \alpha_0 \lambda_1 - 2a\alpha_0 \lambda_1 + \alpha_1 \lambda_1 e^{2\kappa\tau} + \alpha_3 \lambda_1 e^{2\kappa\sigma} + \frac{\alpha_2 \lambda_1}{2\kappa}\left(e^{2\kappa\tau} - 1\right) + \frac{\alpha_4 \lambda_1}{2\kappa}\left(e^{2\kappa\sigma} - 1\right),
$$

$$
\Omega_{12}^1 = 2\lambda_1 \kappa \alpha_0 p - \lambda_1 a\alpha_0 p, \qquad \Omega_{22}^1 = 2\lambda_1 \kappa \alpha_0 p^2 - \alpha_1 (1 - \mu_1),
$$

$$
\Omega_{11}^2 = 2\kappa \alpha_0 \lambda_2 - 2a\alpha_0 \lambda_2 + \alpha_1 \lambda_2 e^{2\kappa\tau} + \alpha_3 \lambda_2 e^{2\kappa\sigma} + \frac{\alpha_2 \lambda_2}{2\kappa}\left(e^{2\kappa\tau} - 1\right) + \frac{\alpha_4 \lambda_2}{2\kappa}\left(e^{2\kappa\sigma} - 1\right),
$$

$$
\Omega_{12}^2 = 2\lambda_2 \kappa \alpha_0 p - \lambda_2 a\alpha_0 p, \qquad \Omega_{22}^2 = 2\lambda_2 \kappa \alpha_0 p^2 - \lambda_2 \alpha_1 (1 - \mu_1),
$$

$$
\lambda_1 = \frac{\tau}{\tau + \sigma}, \qquad \lambda_2 = \frac{\sigma}{\tau + \sigma}.
$$

Proof Consider the Lyapunov functional defined by

$$
V(t, x(t)) = e^{2\kappa t} \alpha_0 \left[x(t) + px(t - \tau(t)) \right]^2
$$

$$
+ \alpha_1 \int_{t-\tau(t)}^t e^{2\kappa(s+\tau)} x^2(s)\, ds + \alpha_2 \int_{-\tau}^0 \int_{t+\theta}^t e^{2\kappa(s-\theta)} x^2(s)\, ds\, d\theta
$$

$$
+ \alpha_3 \int_{t-\sigma(t)}^t e^{2\kappa(s+\sigma)} \tanh^2 x(s)\, ds + \alpha_4 \int_{-\sigma}^0 \int_{t+\theta}^t e^{2\kappa(s-\theta)} \tanh^2 x(s)\, ds\, d\theta, \quad (4)
$$

where α_i ($i = 0, 1, 2, 3, 4$) are positive scalars to be chosen later.

The derivative of $V(t, x(t))$ along the solution of equation (1) is determined by

$$
\frac{dV(t, x(t))}{dt}
$$

$$
= e^{2\kappa t} \bigg\{ 2\kappa \alpha_0 \left[x(t) + px(t - \tau(t)) \right]^2
$$

$$
+ 2\alpha_0 \left[x(t) + px(t - \tau(t)) \right] \left[-ax(t) + b\tanh x(t - \sigma(t)) \right]
$$

$$
+ \alpha_1 e^{2\kappa\tau} x^2(t) - \alpha_1 \left(1 - \tau'(t)\right) e^{2k(\tau - \tau(t))} x^2(t - \tau(t))
$$

$$
+ \frac{\alpha_2}{2\kappa}\left(e^{2\kappa\tau} - 1\right) x^2(t) - \alpha_2 \int_{t-\tau}^t x^2(s)\, ds
$$

$$
+ \alpha_3 e^{2\kappa\sigma} \tanh^2 x(t) - \alpha_3 \left(1 - \sigma'(t)\right) e^{2k(\sigma - \sigma(t))} \tanh^2 x(t - \sigma(t))
$$

$$
+ \frac{\alpha_4}{2\kappa}\left(e^{2\kappa\sigma} - 1\right) \tanh^2 x(t) - \alpha_4 \int_{t-\tau}^t \tanh^2 x(s)\, ds \bigg\}
$$

$$
\leq e^{2\kappa t} \bigg\{ \left[2\kappa\alpha_0 - 2\alpha_0 a + \alpha_1 e^{2\kappa\tau} + \frac{\alpha_2}{2\kappa}\left(e^{2\kappa\tau} - 1\right) \right] x^2(t)
$$

$$+ [4\kappa\alpha_0 p - 2\alpha_0 ap]x(t)x(t - \tau(t))$$

$$+ 2\alpha_0 bx(t)\tanh x(t - \sigma(t)) + [2\kappa\alpha_0 p^2 - \alpha_1(1 - \mu_1)]x^2(t - \tau(t))$$

$$+ 2\alpha_0 pbx(t - \tau(t))\tanh x(t - \sigma(t)) + \left[\alpha_3 e^{2\kappa\sigma} + \frac{\alpha_4}{2\kappa}(e^{2\kappa\sigma} - 1)\right]\tanh^2 x(t)$$

$$- \alpha_3(1 - \mu_2)\tanh^2 x(t - \sigma(t)) - \alpha_2 \int_{t-\tau}^{t} x^2(s)\,ds - \alpha_4 \int_{t-\sigma}^{t} \tanh^2 x(s)\,ds \Bigg\}. \tag{5}$$

The fact that $\tanh^2 x(t) \le x^2(t)$ implies

$$\left[\alpha_3 e^{2\kappa\sigma} + \frac{\alpha_4}{2\kappa}(e^{2\kappa\sigma} - 1)\right]\tanh^2 x(t) \le \left[\alpha_3 e^{2\kappa\sigma} + \frac{\alpha_4}{2\kappa}(e^{2\kappa\sigma} - 1)\right]x^2(t). \tag{6}$$

Substituting (6) into (5), we have

$$\frac{dV(t,x(t))}{dt} \le \frac{1}{\tau}\int_{t-\tau}^{t} \xi_1^T(t,s)\Omega_1\xi_1(t,s)\,ds + \frac{1}{\sigma}\int_{t-\sigma}^{t} \xi_2^T(t,s)\Omega_2\xi_2(t,s)\,ds, \tag{7}$$

where $\xi_1(t,s) = [x^T(t), x^T(t - \tau(t)), \tanh^T x(t - \sigma(t)), x^T(s)]^T$ and $\xi_2(t,s) = [x^T(t), x^T(t - \tau(t)), \tanh^T x(t - \sigma(t)), \tanh^T x(s)]^T$.

From (3), we have $\frac{dV(t,x(t))}{dt} < 0$, which implies $V(t,x(t)) \le V(0,x(0))$. And from the definition of the Lypunov function $V(t,x(t))$, we have

$$V(0,x(0)) = \alpha_0\big[x(0) + px(-\tau(0))\big]^2$$

$$+ \alpha_1 \int_{-\tau(0)}^{0} e^{2\kappa(s+\tau)}x^2(s)\,ds + \alpha_2 \int_{-\tau}^{0}\int_{\theta}^{0} e^{2\kappa(s-\theta)}x^2(s)\,ds\,d\theta$$

$$+ \alpha_3 \int_{-\sigma(0)}^{0} e^{2\kappa(s+\sigma)}\tanh^2 x(s)\,ds + \alpha_4 \int_{-\sigma}^{0}\int_{\theta}^{0} e^{2\kappa(s-\theta)}\tanh^2 x(\theta)\,d\theta\,ds$$

$$\le \left[4\alpha_0 + (\alpha_1 + \alpha_3)re^{2\kappa r} + (\alpha_2 + \alpha_4)\frac{e^{2\kappa r}}{4\kappa^2}\right]\sup_{\theta\in[-r,0]}\big|\phi(\theta)\big|^2$$

$$\equiv M.$$

So, we obtain

$$\big|x(t) + px(t - \tau(t))\big|^2 \le M_1 e^{-2\kappa t}, \tag{8}$$

where $M_1 = \frac{M}{\alpha_0} > 0$. For $\forall \varepsilon \in (0, \min\{2\kappa, -\frac{2}{r}\log|p|\})$ and $\nu > 0$, the fundamental inequality $xy \le \nu x^2 + \frac{1}{\nu}y^2$ for any $x, y \in R$ implies

$$e^{\varepsilon t}\big|x(t)\big|^2 \le (1 + \nu)e^{\varepsilon t}\big|x(t) + px(t - \tau(t))\big|^2 + \frac{1 + \nu}{\nu}e^{\varepsilon t}\big|px(t - \tau(t))\big|^2$$

$$\le (1 + \nu)M_1 + \frac{1 + \nu}{\nu}|p|^2 e^{\varepsilon r}e^{\varepsilon(t-\tau(t))}\big|x(t - \tau(t))\big|^2.$$

And from $\varepsilon \in (0, \min\{2\kappa, -\frac{2}{r}\log|p|\})$, we have $|p|^2 e^{\varepsilon r} < 1$. Thus, as $\nu > 0$ is chosen sufficiently large,

$$\epsilon = \frac{|p|^2(1 + \nu)e^{\varepsilon r}}{\nu} < 1.$$

Therefore, we have

$$e^{\varepsilon t}\left|x(t)\right|^2 \le (1+\nu)M_1 + \epsilon e^{\varepsilon(t-\tau(t))}\left|x\big(t-\tau(t)\big)\right|^2. \tag{9}$$

About (9), for $\forall T \ge 0$,

$$\sup_{0\le t\le T}\left\{e^{\varepsilon t}\left|x(t)\right|^2\right\} \le (1+\nu)M_1 + \epsilon \sup_{\theta\in[-r,0]}\left|\phi(\theta)\right|^2 + \epsilon \sup_{0\le t\le T}\left\{e^{\varepsilon t}\left|x(t)\right|^2\right\}.$$

Consequently,

$$\sup_{0\le t\le T}\left\{e^{\varepsilon t}\left|x(t)\right|^2\right\} \le \frac{(1+\nu)M_1 + \epsilon \sup_{\theta\in[-r,0]}|\phi(\theta)|^2}{1-\epsilon}. \tag{10}$$

As $T \to +\infty$, it follows from (10) that

$$\sup_{0\le t<+\infty}\left[e^{\varepsilon t}\left|x(t)\right|^2\right] \le \frac{(1+\nu)M_1 + \epsilon \sup_{\theta\in[-r,0]}|\phi(\theta)|^2}{1-\epsilon},$$

that is,

$$\left|x(t)\right| \le M_2 e^{-\alpha t},$$

where $M_2 = \sqrt{\frac{(1+\nu)M_1+\epsilon\sup_{\theta\in[-r,0]}|\phi(\theta)|^2}{1-\epsilon}} > 0$ and $\alpha = \frac{\varepsilon}{2} > 0$. The proof of this theorem is completed. $\qquad\square$

When $\tau(t) \equiv \tau, \sigma(t) \equiv \sigma$, we can easily derive the following corollary.

Corollary 2 *For a given positive constant $\kappa > 0$, the zero solution of equation* (2) *is globally exponentially stable if there exist some positive scalars: α_0, α_1, α_2, α_3, α_4, such that the following linear matrix inequalities* (*LMIs*) *hold*:

$$\Omega_1 = \begin{bmatrix} \Omega_{11}^1 & \Omega_{12}^1 & \lambda_1\alpha_0 b & 0 \\ * & \Omega_{22}^1 & \lambda_1\alpha_0 pb & 0 \\ * & * & -\lambda_1\alpha_3 & 0 \\ * & * & * & -\alpha_2\tau \end{bmatrix} < 0,$$

$$\Omega_2 = \begin{bmatrix} \Omega_{11}^2 & \Omega_{12}^2 & \lambda_2\alpha_0 b & 0 \\ * & \Omega_{22}^2 & \lambda_2\alpha_0 pb & 0 \\ * & * & -\alpha_3\lambda_2 & 0 \\ * & * & * & -\alpha_4\sigma \end{bmatrix} < 0,$$

where

$$\Omega_{12}^1 = 2\lambda_1\kappa\alpha_0 p - \lambda_1 a\alpha_0 p, \qquad \Omega_{22}^1 = 2\lambda_1\kappa\alpha_0 p^2 - \lambda_1\alpha_1,$$

$$\Omega_{12}^2 = 2\lambda_2\kappa\alpha_0 p - \lambda_2 a\alpha_0 p, \qquad \Omega_{22}^2 = 2\lambda_2\kappa\alpha_0 p^2 - \lambda_2\alpha_1,$$

and Ω_{11}^1, Ω_{11}^2, λ_1, λ_2 are given in Theorem 1.

Remark 1 The proofs of Theorem 1 and Corollary 2 are completed by utilizing the technique involved in [19], so that the mode-transformation technique and the bounding technique are not employed. Although one LMI-based sufficient condition ensuring the exponential stability for equation (2) has been obtained in [16], this condition is more conservative since the mode-transformation technique and the bounding technique are both used, and the obtained result has narrow applications since the condition

$$|p| + (\sigma - \tau)|b| < 1, \tag{11}$$

must be imposed. Besides, note that Theorem 4 in [16] involves seven decision variables, while Corollary 2 involves five decision variables. Thus, Corollary 2 needs fewer decision variables than Theorem 4 in [16]. What is more, this restrictive condition (11) is removed in this paper. Thus, our LMIs-based sufficient conditions are less conservative than those provided in [16], which is shown by Example 1 and Example 2 in Section 3. And the technique employed in this paper is different from the previous ones introduced in [1, 3, 7–11, 16, 17].

Remark 2 Although the delay-independent sufficient condition for the global exponential stability of equation (2) has been obtained, the technique used in [8] is only suitable for constant delay, not for time-varying delays. So, our result can complement the result in [8]. Besides, in [8], the delay-independent sufficient condition for the global exponential stability of equation (2) has been given in the form

$$a(1 - |p|) > b(1 + |p|). \tag{12}$$

Remark 3 If $\kappa = 0$, the criteria about the global asymptotical stability for equation (1) are presented as follows:

Corollary 3 *The zero solution of equation* (1) *is globally asymptotically stable if there exist some positive scalars*: $\alpha_0, \alpha_1, \alpha_2, \alpha_3$ *and* α_4, *such that the following linear matrix inequalities* (*LMIs*) *hold*:

$$
\Omega_1 = \begin{bmatrix} \Omega_{11}^1 & \Omega_{12}^1 & \lambda_1\alpha_0 b & 0 \\ * & \Omega_{22}^1 & \lambda_1\alpha_0 pb & 0 \\ * & * & -\lambda_1\alpha_3(1 - \mu_2) & 0 \\ * & * & * & -\alpha_2\tau \end{bmatrix} < 0,
$$

$$
\Omega_2 = \begin{bmatrix} \Omega_{11}^2 & \Omega_{12}^2 & \lambda_2\alpha_0 b & 0 \\ * & \Omega_{22}^2 & \lambda_2\alpha_0 pb & 0 \\ * & * & -\lambda_2\alpha_3(1 - \mu_2) & 0 \\ * & * & * & -\alpha_4\sigma \end{bmatrix} < 0, \tag{13}
$$

where

$$\Omega_{11}^1 = -2\lambda_1 a\alpha_0 + \lambda_1\alpha_1 + \lambda_1\alpha_2\tau + \lambda_1\alpha_3 + \lambda_1\alpha_4\sigma,$$

$$\Omega_{12}^1 = -\lambda_1 a\alpha_0 p, \qquad \Omega_{22}^1 = -\lambda_1\alpha_1(1 - \mu_1),$$

Table 1 Comparisons of maximum allowed σ for Example 1 when $\tau = 0.1$

σ	[3]	[7]	[9]	[9]	[10]	[11]	[16]	[18]	This paper
A. S.	2.22	10^7	2.07	2.32	1.90	0.444	2.32	10^{21}	1.38×10^{21}
E. S. ($\kappa = 0.0038$)	-	-	-	-	-	-	1.947	-	175.289

Notations: A. S. stands for asymptotically stable, E. S. stands for exponentially stable.

$$\Omega_{11}^2 = -2\lambda_2 a\alpha_0 + \lambda_2\alpha_1 + \lambda_2\alpha_2\tau + \lambda_2\alpha_3 + \lambda_2\alpha_4\sigma,$$

$$\Omega_{12}^2 = -\lambda_2 a\alpha_0 p, \quad \Omega_{22}^2 = -\lambda_2\alpha_1(1 - \mu_1),$$

λ_i $(i = 1, 2)$ *are given in Theorem* 1.

3 Three illustrative examples

In this section, three illustrative examples are given to show the effectiveness of our result.

Example 1 Consider the following equation in [3, 7, 9–11, 16, 18]:

$$\frac{d}{dt}\big[x(t) + 0.2x(t - 0.1)\big] = -0.6x(t) + 0.3\tanh x(t - \sigma). \tag{14}$$

By Corollary 2, if $\kappa = 0.0038$ is given, the upper bound of the time-delay σ which guarantees the exponential stability of equation (14) is 175.289. Obviously, when $\sigma = 175.289$, the two inequalities (11) and (12) do not hold and both criteria given in [8, 16] have a narrow application. And, by Corollary 3, the upper bound of time-varying delay $\sigma(t)$ for the global asymptotic stability of equation (14) is 1.38×10^{21}. The comparison results of the maximum allowable delay bounds σ in [3, 7, 9–11, 16, 18] and our result are listed in Table 1. Obviously, our result is less conservative than those in [3, 7, 9–11, 16, 18]. By solving the LMIs (13) in Corollary 3, with respect to $\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4$, we obtain a solution

$$\alpha_0 = 4.7784 \times e^8, \qquad \alpha_1 = 1.0407 \times e^8, \qquad \alpha_2 = 2.7196 \times e^8,$$

$$\alpha_3 = 1.6709 \times e^8, \qquad \alpha_4 = 5.9603 \times e^{-14},$$

when the maximum value of σ is 1.38×10^{21}.

Example 2 Consider the following equation in [1, 3, 5, 7–10, 16, 18]:

$$\frac{d}{dt}\big[x(t) + 0.35x(t - 0.5)\big] = -1.5x(t) + b\tanh x(t - 0.5). \tag{15}$$

By Corollary 2, if $\kappa = 0.177$ is given, we obtain that the upper bound of b about the global exponential stability of equation (15) is 1.0929. And by Corollary 3, the upper bound of b ensuring the global asymptotic stability of equation (15) is up to 1.405. Obviously, it is seen from Table 2 that our result is better than those given in [1, 3, 5, 7–10, 16, 18]. By solving the LMIs (13) with respect to $\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4$, if $\kappa = 0.177$, we obtain a solution

$$\alpha_0 = 5.3090 \times e^3, \qquad \alpha_1 = 1.0908 \times e^3, \qquad \alpha_2 = 0.3857,$$

$$\alpha_3 = 4.7933 \times e^3, \qquad \alpha_4 = 0.3857,$$

Table 2 The upper bound of b for Example 2 when $\sigma = \tau = 0.5$

b	[1]	[18]	[3]	[5]	[7]	[8]	[9]	[10]	[16]	This paper
A. S.	0.318	1.346	0.889	0.424	1.49	0.699	1.405	0.422	1.405	1.405
E. S. ($\kappa = 0.177$)	-	-	-	-	-	0.722	-	-	0.478	1.092
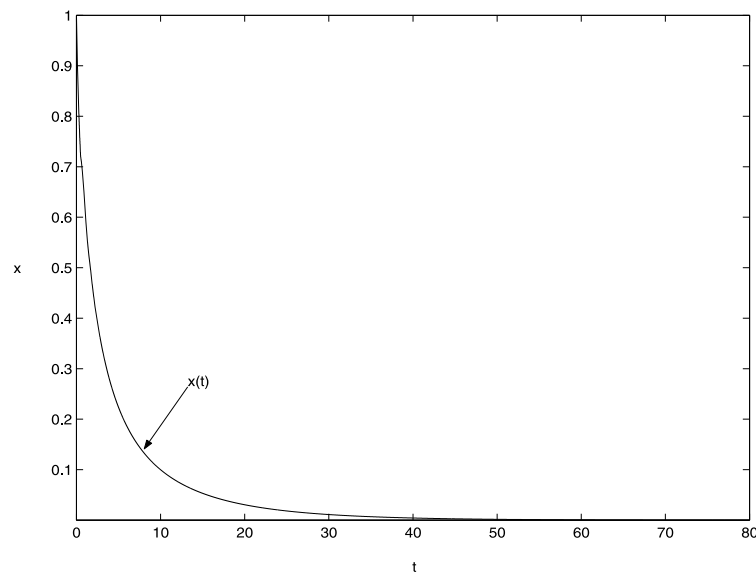
Figure 1 The trajectory of the solution to such a system in Example 2.

when the maximum value of b is 1.0929. When $b = 1.0929$, the criterion about the global exponential stability of equation (15) is not obtained in [8] since the inequality (12) is not satisfied. It is easily seen that our result is less conservative than one in [8]. The simulation for the trajectory of the solution to such a system when $\sigma = \tau = 0.5$ and $b = 1.405$ is given in Figure 1.

Example 3 Consider the following equation with time-varying delays [18]:

$$\frac{d}{dt}\big[x(t) + 0.2x\big(t - \tau(t)\big)\big] = -0.6x(t) + 0.5\tanh x\big(t - \sigma(t)\big), \tag{16}$$

where $\tau(t) = \frac{\sin^2(t)}{10}$ and $\mu_2 = 0.2$. By using Corollary 3, the upper bound of time-varying delay $\sigma(t)$ for asymptotic stability of equation (16) is 1.285×10^{25}. But, by virtue of Theorem 1 in [18], the maximin allowable bound of time-varying delay $\sigma(t)$ for asymptotic stability of this equation is 5.000×10^{24}. So, Corollary 3 is better than one in [18]. When $\sigma = 1.285 \times 10^{25}$, by the Matlab LMIs Control Toolbox, a solution to the LMIs (13) in Corollary 3 can be obtained as

$$\alpha_0 = 4.6259 \times e^8, \qquad \alpha_1 = 2.5166 \times e^7, \qquad \alpha_2 = 4.0187 \times e^7,$$

$$\alpha_3 = 2.4431 \times e^8, \qquad \alpha_4 = 3.8420 \times e^{-19}.$$

Obviously, the result provided in [16] is infeasible. And even if the delays $\tau(t)$ and $\sigma(t)$ are the constant delays, the zero solution of equation (16) is not globally exponentially stable according to the criterion given in [8] since the inequality (12) is not satisfied. Thus, our result can also complement that in [8, 16].

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