

RESEARCH

Open Access

Sets of generalized \mathcal{H}_2 exponential stability criteria for switched multilayer dynamic neural networks

Choon Ki Ahn^{1*} and Young Sam Lee²

*Correspondence:
hironaka@korea.ac.kr
¹School of Electrical Engineering,
Korea University, Anam-dong,
Seongbuk-Gu, Seoul, 136-701, Korea
Full list of author information is
available at the end of the article

Abstract

This paper investigates new sets of generalized \mathcal{H}_2 exponential stability criteria for switched multilayer dynamic neural networks. These sets of sufficient stability criteria in forms of linear matrix inequality (LMI) and matrix norm are presented, under which switched multilayer dynamic neural networks reduce the effect of external input to a predefined level. The proposed sets of criteria also guarantee exponential stability for switched multilayer dynamic neural networks without external input.

1 Introduction

Switched systems are a class of hybrid systems consisting of a family of continuous (or discrete) time subsystems and a logical rule that orchestrates the switching between these subsystems. Switched systems have been extensively researched, and several efforts have been focused on the analysis of switched systems [1, 2]. Recently, switched recurrent neural networks were introduced to represent some complex nonlinear systems efficiently by integrating the theory of switched systems with recurrent neural networks [3–6]. Switched dynamic neural networks have found applications in the field of artificial intelligence and high speed signal processing [7, 8]. In [3–6], some stability criteria for switched dynamic neural networks were studied.

There always exist noise disturbances and model uncertainties in real physical systems. Recently, this has led to an interest in a generalized \mathcal{H}_2 approach [9–19]. The generalized \mathcal{H}_2 approach has been known as a significant concept to examine the stability of various nonlinear dynamical systems. Here, we have the following natural question: Can we obtain a generalized \mathcal{H}_2 stability criterion for switched dynamic neural networks. This paper provides an answer to this question. To the best of the authors' knowledge, the generalized \mathcal{H}_2 analysis of switched dynamic neural networks has not yet been studied in the literature.

In this paper, we propose new sets of generalized \mathcal{H}_2 exponential stability criteria for switched multilayer dynamic neural networks. The sets of conditions proposed in this paper are a new contribution to the stability evaluation of switched neural networks. The proposed sets of sufficient stability criteria in forms of linear matrix inequality (LMI) and matrix norm guarantee that switched multilayer dynamic neural networks reduce the effect of external input to a predefined level. This paper is organized as follows. In Section 2, new sets of generalized \mathcal{H}_2 exponential stability criteria are derived. Finally, conclusions are presented in Section 3.

2 New sets of generalized \mathcal{H}_2 exponential stability criteria

Consider the following model of switched multilayer neural networks [3]:

$$\dot{x}(t) = A_\alpha x(t) + W_\alpha \phi(V_\alpha x(t)) + J(t), \quad (1)$$

$$z(t) = H_\alpha x(t), \quad (2)$$

where $x(t) = [x_1(t) \ \cdots \ x_n(t)]^T \in R^n$ is the state vector, $z(t) \in R^p$ is a linear combination of the states, $A = \text{diag}\{-a_1, \dots, -a_n\} \in R^{n \times n}$ ($a_k > 0$, $k = 1, \dots, n$) is the self-feedback matrix, $W \in R^{n \times n}$ and $V \in R^{n \times n}$ are the connection weight matrices, $\phi(x(t)) = [\phi_1(x(t)) \ \cdots \ \phi_n(x(t))]^T : R^n \rightarrow R^n$ is the nonlinear function vector satisfying the global Lipschitz condition with Lipschitz constant $L_\phi > 0$, $J(t) \in R^n$ is an external input vector, and $H \in R^{p \times n}$ is a known constant matrix. α is a switching signal which takes its values in the finite set $\mathcal{I} = \{1, 2, \dots, N\}$. The matrices $(A_\alpha, W_\alpha, V_\alpha, H_\alpha)$ are allowed to take values, at an arbitrary time, in the finite set $\{(A_1, W_1, V_1, H_1), \dots, (A_N, W_N, V_N, H_N)\}$. Throughout this paper, we assume that the switching rule α is not known a priori and its instantaneous value is available in real time. Define the indicator function $\xi(t) = (\xi_1(t), \xi_2(t), \dots, \xi_N(t))^T$, where

$$\xi_i(t) = \begin{cases} 1, & \text{when the switched system is described by the } i\text{th mode } (A_i, W_i, V_i, H_i), \\ 0, & \text{otherwise,} \end{cases}$$

with $i = 1, \dots, N$. By using this indicator function, the model of the switched multilayer neural networks (1)-(2) can be written as

$$\dot{x}(t) = \sum_{i=1}^N \xi_i(t) [A_i x(t) + W_i \phi(V_i x(t)) + J(t)], \quad (3)$$

$$z(t) = \sum_{i=1}^N \xi_i(t) H_i x(t), \quad (4)$$

where $\sum_{i=1}^N \xi_i(t) = 1$ is satisfied under any switching rules. Let $\gamma > 0$ be a predefined level of disturbance attenuation. In this paper, for a given $\kappa > 0$, we derive sets of criteria such that the switched multilayer neural network (3)-(4) with $J(t) = 0$ is exponentially stable ($\|x(t)\| \leq \Upsilon \exp(-\kappa t)$, where $\Upsilon > 0$) and

$$\sup_{t \geq 0} \{ \exp(\kappa t) z^T(t) z(t) \} < \gamma^2 \int_0^\infty \exp(\kappa t) J^T(t) J(t) dt, \quad (5)$$

under zero-initial conditions for all nonzero $J(t) \in L_2[0, \infty)$, where $L_2[0, \infty)$ is the space of square integrable vector functions over $[0, \infty)$.

A set of generalized \mathcal{H}_2 exponential stability criterion of the switched multilayer neural network (3)-(4) is derived in the following theorem.

Theorem 1 For given $\gamma > 0$ and $\kappa > 0$, the switched multilayer neural network (3)-(4) is generalized \mathcal{H}_2 exponentially stable if

$$\|W_i\| < \frac{\sqrt{k_i - 2\|P\|^2 - \kappa\|P\|^2}}{L_\phi\|V_i\|}, \quad (6)$$

$$\|V_i\| \neq 0, \quad (7)$$

$$\|P\| < \sqrt{\frac{k_i}{2+\kappa}}, \quad k_i > 0, P = P^T > 0, \quad (8)$$

$$\|H_i\| \leq \gamma \sqrt{\lambda_{\min}(P)}, \quad (9)$$

for $i = 1, \dots, N$, where $\lambda_{\min}(\cdot)$ is the minimum eigenvalue of the matrix and P satisfies the Lyapunov inequality $A_i^T P + P A_i < -k_i I$.

Proof We consider the Lyapunov function $V(t) = \exp(\kappa t)x^T(t)Px(t)$. The time derivative of the function along the trajectory of (3) satisfies

$$\dot{V}(t) < \sum_{i=1}^N \xi_i(t) \exp(\kappa t) \{-x^T(t)[k_i - \kappa P]x(t) + 2x^T(t)PW_i\phi(V_i x(t)) + 2x^T(t)PJ(t)\}. \quad (10)$$

Applying Young's inequality [20], we have $2x^T(t)PW_i\phi(V_i x(t)) \leq x^T(t)PPx(t) + \phi^T \times (V_i x(t))W_i^T W_i \phi(V_i x(t)) \leq \|P\|^2 \|x(t)\|^2 + L_\phi^2 \|W_i\|^2 \|V_i\|^2 \|x(t)\|^2$ and $2x^T(t)PJ(t) \leq x^T(t)P \times P^T x(t) + J^T(t)J(t) \leq \|P\|^2 \|x(t)\|^2 + \|J(t)\|^2$. Substituting these inequalities into (10), we have

$$\begin{aligned} \dot{V}(t) &< \sum_{i=1}^N \xi_i(t) \exp(\kappa t) \{-[k_i - (2 + \kappa)\|P\|^2 - L_\phi^2 \|W_i\|^2 \|V_i\|^2] \|x(t)\|^2 + \|J(t)\|^2\} \\ &= - \sum_{i=1}^N \xi_i(t) \exp(\kappa t) [k_i - (2 + \kappa)\|P\|^2 - L_\phi^2 \|W_i\|^2 \|V_i\|^2] \|x(t)\|^2 \\ &\quad + \sum_{i=1}^N \xi_i(t) \exp(\kappa t) \|J(t)\|^2. \end{aligned} \quad (11)$$

If the following condition is satisfied,

$$k_i - (2 + \kappa)\|P\|^2 - L_\phi^2 \|W_i\|^2 \|V_i\|^2 > 0, \quad (12)$$

for $i = 1, \dots, N$, we have

$$\begin{aligned} \dot{V}(t) &< \sum_{i=1}^N \xi_i(t) \exp(\kappa t) \|J(t)\|^2 \\ &= \exp(\kappa t) \|J(t)\|^2. \end{aligned} \quad (13)$$

The following inequalities

$$\|W_i\|^2 < \frac{k_i - (2 + \kappa)\|P\|^2}{L_\phi^2 \|V_i\|^2}, \quad \|P\|^2 < \frac{k_i}{2 + \kappa}, \quad (14)$$

for $i = 1, \dots, N$, imply the condition (12). Thus, we obtain (6) and (7). Under the zero-initial condition, we have $V(t)|_{t=0} = 0$ and $V(t) \geq 0$. Define

$$\Phi(t) = V(t) - \int_0^t \exp(\kappa \sigma) J^T(\sigma) J(\sigma) d\sigma. \quad (15)$$

Then, for any nonzero $J(t)$, we obtain

$$\begin{aligned}\Phi(t) &= V(t) - V(t)|_{t=0} - \int_0^t \exp(\kappa\sigma) J^T(\sigma) J(\sigma) d\sigma \\ &= \int_0^t [\dot{V}(\sigma) - \exp(\kappa\sigma) J^T(\sigma) J(\sigma)] d\sigma.\end{aligned}$$

From (13), we have $\Phi(t) < 0$. It means

$$V(t) < \int_0^t \exp(\kappa\sigma) J^T(\sigma) J(\sigma) d\sigma.$$

The condition (9) implies

$$\begin{aligned}\exp(\kappa t) z^T(t) z(t) &= \sum_{i=1}^N \xi_i(t) \exp(\kappa t) x^T(t) H_i^T H_i x(t) \\ &\leq \sum_{i=1}^N \xi_i(t) \exp(\kappa t) \|H_i\|^2 \|x(t)\|^2 \\ &\leq \gamma^2 \sum_{i=1}^N \xi_i(t) \exp(\kappa t) \lambda_{\min}(P) \|x(t)\|^2 \\ &\leq \gamma^2 \sum_{i=1}^N \xi_i(t) \exp(\kappa t) x^T(t) P x(t) \\ &= \gamma^2 V(t) \\ &< \gamma^2 \int_0^t \exp(\kappa\sigma) J^T(\sigma) J(\sigma) d\sigma \\ &\leq \gamma^2 \int_0^\infty \exp(\kappa\sigma) J^T(\sigma) J(\sigma) d\sigma.\end{aligned}\tag{16}$$

Taking the supremum over $t > 0$ leads to (5). This completes the proof. \square

Corollary 1 *When $J(t) = 0$, the conditions (6)-(9) ensure that the switched multilayer neural network (3)-(4) is exponentially stable.*

Proof When $J(t) = 0$, from (13), $\dot{V}(t) < 0$ for all $x(t) \neq 0$. Thus, for any $t \geq 0$, it implies that

$$\exp(\kappa t) \|x(t)\|^2 \lambda_{\min}(P) \leq \exp(\kappa t) x^T(t) P x(t) = V(t) < V(0) = x^T(0) P x(0).\tag{17}$$

Finally, we have

$$\|x(t)\| < \sqrt{\frac{x^T(0) P x(0)}{\lambda_{\min}(P)}} \exp\left(-\frac{\kappa}{2} t\right).\tag{18}$$

This completes the proof. \square

In the next theorem, we find a new set of LMI criteria for the generalized \mathcal{H}_2 exponential stability of the switched multilayer neural network (3)-(4). This set of LMI criteria can be facilitated readily via standard numerical algorithms [21, 22].

Theorem 2 For given level $\gamma > 0$ and $\kappa > 0$, the switched multilayer neural network (3)-(4) is generalized \mathcal{H}_2 exponentially stable if there exist a positive symmetric matrix P and a positive scalar ϵ such that

$$\begin{bmatrix} A_i^T P + PA_i + \kappa P + \epsilon L_\phi^2 V_i^T V_i & P W_i & P \\ W_i^T P & -\epsilon I & 0 \\ P & 0 & -I \end{bmatrix} < 0, \quad (19)$$

$$\begin{bmatrix} P & H_i^T \\ H_i & \gamma^2 I \end{bmatrix} > 0, \quad (20)$$

for $i = 1, \dots, N$.

Proof Consider the Lyapunov function $V(t) = \exp(\kappa t) x^T(t) P x(t)$. Applying Young's inequality [20], we have $\epsilon [L_\phi^2 x^T(t) V_i^T V_i x(t) - \phi^T(V_i x(t)) \phi(V_i x(t))] \geq 0$. By using this inequality, the time derivative of $V(t)$ along the trajectory of (3) is

$$\begin{aligned} \dot{V}(t) &= \sum_{i=1}^N \xi_i(t) \exp(\kappa t) \{ x^T(t) [A_i^T P + PA_i + \kappa P] x(t) \\ &\quad + 2x^T(t) P W_i \phi(V_i x(t)) + 2x^T(t) P J(t) \} \\ &\leq \sum_{i=1}^N \xi_i(t) \exp(\kappa t) \{ x^T(t) [A_i^T P + PA_i + \kappa P] x(t) \\ &\quad + 2x^T(t) P W_i \phi(V_i x(t)) + 2x^T(t) P J(t) \\ &\quad + \epsilon [L_\phi^2 x^T(t) V_i^T V_i x(t) - \phi^T(V_i x(t)) \phi(V_i x(t))] \} \\ &= \sum_{i=1}^N \xi_i(t) \exp(\kappa t) \begin{bmatrix} x(t) \\ \phi(V_i x(t)) \\ J(t) \end{bmatrix}^T \begin{bmatrix} A_i^T P + PA_i + \kappa P + \epsilon L_\phi^2 V_i^T V_i & P W_i & P \\ W_i^T P & -\epsilon I & 0 \\ P & 0 & -I \end{bmatrix} \\ &\quad \times \begin{bmatrix} x(t) \\ \phi(V_i x(t)) \\ J(t) \end{bmatrix} + \sum_{i=1}^N \xi_i(t) \exp(\kappa t) J^T(t) J(t). \end{aligned} \quad (21)$$

If the LMI (19) is satisfied, we have

$$\begin{aligned} \dot{V}(t) &< \sum_{i=1}^N \xi_i(t) \exp(\kappa t) J^T(t) J(t) \\ &= \exp(\kappa t) J^T(t) J(t). \end{aligned} \quad (22)$$

Under the zero-initial condition, one has $V(t)|_{t=0} = 0$ and $V(t) \geq 0$. Define

$$\Phi(t) = V(t) - \int_0^t \exp(\kappa \sigma) J^T(\sigma) J(\sigma) d\sigma. \quad (23)$$

Then, for any nonzero $J(t)$, we obtain

$$\begin{aligned}\Phi(t) &= V(t) - V(t)|_{t=0} - \int_0^t \exp(\kappa\sigma) J^T(\sigma) J(\sigma) d\sigma \\ &= \int_0^t [\dot{V}(\sigma) - \exp(\kappa\sigma) J^T(\sigma) J(\sigma)] d\sigma.\end{aligned}$$

From (22), we have $\Phi(t) < 0$. It means

$$V(t) < \int_0^t \exp(\kappa\sigma) J^T(\sigma) J(\sigma) d\sigma.$$

The LMI (20) implies

$$\begin{aligned}\exp(\kappa t) z^T(t) z(t) &= \sum_{i=1}^N \xi_i(t) \exp(\kappa t) x^T(t) H_i^T H_i x(t) \\ &< \gamma^2 \sum_{i=1}^N \xi_i(t) \exp(\kappa t) x^T(t) P x(t) \\ &= \gamma^2 V(t) \\ &< \gamma^2 \int_0^t \exp(\kappa\sigma) J^T(\sigma) J(\sigma) d\sigma \\ &\leq \gamma^2 \int_0^\infty \exp(\kappa\sigma) J^T(\sigma) J(\sigma) d\sigma.\end{aligned}\tag{24}$$

Taking the supremum over $t > 0$ leads to (5). This completes the proof. \square

Corollary 2 When $J(t) = 0$, a set of LMI conditions (19)-(20) ensure that the switched multilayer neural network (3)-(4) is exponentially stable.

Proof When $J(t) = 0$, from (22), we have

$$\dot{V}(t) < 0, \quad \forall x(t) \neq 0.\tag{25}$$

Thus, for any $t \geq 0$, it implies that

$$\exp(\kappa t) \|x(t)\|^2_{\lambda_{\min}(P)} \leq \exp(\kappa t) x^T(t) P x(t) = V(t) < V(0) = x^T(0) P x(0).\tag{26}$$

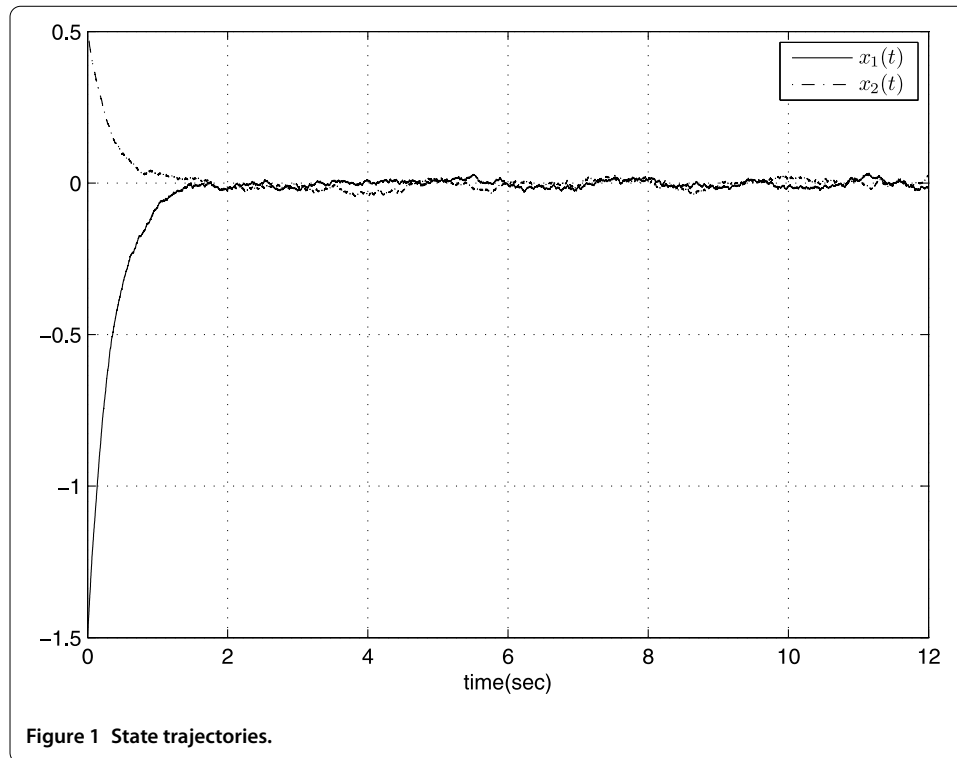
Finally, we have

$$\|x(t)\| < \sqrt{\frac{x^T(0) P x(0)}{\lambda_{\min}(P)}} \exp\left(-\frac{\kappa}{2} t\right).\tag{27}$$

This completes the proof. \square

Example 1 Consider the switched neural network (3)-(4), where

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}, \quad \phi(x(t)) = \begin{bmatrix} \tanh(x_1(t)) \\ \tanh(x_2(t)) \end{bmatrix}, \quad J(t) = \begin{bmatrix} J_1(t) \\ J_2(t) \end{bmatrix},$$



$$\begin{aligned}
 A_1 &= \begin{bmatrix} -2.5 & 0 \\ 0 & -3.1 \end{bmatrix}, & A_2 &= \begin{bmatrix} -3.1 & 0 \\ 0 & -2.32 \end{bmatrix}, \\
 W_1 &= \begin{bmatrix} -1 & 0.4 \\ 0 & -0.1 \end{bmatrix}, & W_2 &= \begin{bmatrix} 1 & -0.8 \\ 0.4 & 0.5 \end{bmatrix}, \\
 V_1 &= \begin{bmatrix} 0.1 & -0.7 \\ -0.5 & 0 \end{bmatrix}, & V_2 &= \begin{bmatrix} -0.1 & 0.3 \\ 0 & -0.5 \end{bmatrix}, \\
 H_1 &= \begin{bmatrix} 1 & 0 \end{bmatrix}, & H_2 &= \begin{bmatrix} 0 & 1 \end{bmatrix}, & N &= 2, & \kappa &= 0.1.
 \end{aligned}$$

Applying Theorem 2 with $\gamma = 0.52$, we obtain

$$P = \begin{bmatrix} 3.7405 & 0.0474 \\ 0.0474 & 3.7059 \end{bmatrix}, \quad \epsilon = 6.2769.$$

The switching signal $\alpha \in \{1, 2\}$ is given by

$$\alpha = \begin{cases} 1, & 2k \leq t \leq 2k+1, k \in \mathbb{Z}, \\ 2, & \text{otherwise,} \end{cases}$$

where \mathbb{Z} is the whole set of nonnegative integers. Figure 1 shows state trajectories when $x(0) = [-1.5 \ 0.5]^T$ and $J_i(t)$ ($i = 1, 2$) is a white noise.

3 Conclusion

In this paper, we have proposed new sets of generalized \mathcal{H}_2 exponential stability criteria for switched multilayer dynamic neural networks. These sets of sufficient stability

criteria are represented by matrix norm and LMI. The proposed sets of criteria ensured that switched multilayer dynamic neural networks attenuate the effect of external input on the state vector. These sets of criteria also guaranteed exponential stability for switched multilayer dynamic neural networks when there is no external input.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors contributed equally and significantly in writing this paper. All authors read and approved the final manuscript.

Author details

¹School of Electrical Engineering, Korea University, Anam-dong, Seongbuk-Gu, Seoul, 136-701, Korea. ²Department of Electrical Engineering, Inha University, 253 Yonghyun-dong, Nam-gu, Incheon, 402-751, Korea.

Acknowledgements

This research was supported by the MKE (The Ministry of Knowledge Economy), Korea, under the CITRC (Convergence Information Technology Research Center) support program (NIPA-2012-H0401-12-1007) supervised by the NIPA (National IT Industry Promotion Agency).

Received: 26 March 2012 Accepted: 20 July 2012 Published: 3 September 2012

References

1. Lee, S, Kim, T, Lim, J: A new stability analysis of switched systems. *Automatica* **36**(6), 917-922 (2000)
2. Daafouz, J, Riedinger, P, Iung, C: Stability analysis and control synthesis for switched systems: a switched Lyapunov function approach. *IEEE Trans. Autom. Control* **47**(11), 1883-1887 (2002)
3. Huang, H, Qu, Y, Li, H: Robust stability analysis of switched Hopfield neural networks with time-varying delay under uncertainty. *Phys. Lett. A* **345**, 345-354 (2005)
4. Yuan, K, Cao, J, Li, H: Robust stability of switched Cohen-Grossberg neural networks with mixed time-varying delays. *IEEE Trans. Syst. Man Cybern., Part B, Cybern.* **36**(6), 1356-1363 (2006)
5. Li, P, Cao, J: Global stability in switched recurrent neural networks with time-varying delay via nonlinear measure. *Nonlinear Dyn.* **49**(1-2), 295-305 (2007)
6. Lou, X, Cui, B: Delay-dependent criteria for robust stability of uncertain switched Hopfield neural networks. *Int. J. Autom. Comput.* **4**(3), 304-314 (2007)
7. Tsividis, Y: Switched neural networks. US Patent 4873661, 1989
8. Brown, T: Neural networks for switching. *IEEE Commun. Mag.* **27**(11), 72-81 (1989)
9. Grigoriadis, K, Watson, J: Reduced-order \mathcal{H}_∞ and $\mathcal{L}_2 - \mathcal{L}_\infty$ filtering via linear matrix inequalities. *IEEE Trans. Aerosp. Electron. Syst.* **33**, 1326-1338 (1997)
10. Watson, J, Grigoriadis, K: Optimal unbiased filtering via linear matrix inequalities. *Syst. Control Lett.* **35**, 111-118 (1998)
11. Palhares, R, Peres, P: Robust filtering with guaranteed energy-to-peak performance - an LMI approach. *Automatica* **36**, 851-858 (2000)
12. Gao, H, Wang, C: Robust $\mathcal{L}_2 - \mathcal{L}_\infty$ filtering for uncertain systems with multiple time-varying state delays. *IEEE Trans. Circuits Syst. I, Fundam. Theory Appl.* **50**, 594-599 (2003)
13. Gao, H, Wang, C: Delay-dependent robust \mathcal{H}_∞ and $\mathcal{L}_2 - \mathcal{L}_\infty$ filtering for a class of uncertain nonlinear time-delay systems. *IEEE Trans. Autom. Control* **48**, 1661-1666 (2003)
14. Mahmoud, M: Resilient $\mathcal{L}_2 - \mathcal{L}_\infty$ filtering of polytopic systems with state delays. *IET Control Theory Appl.* **1**, 141-154 (2007)
15. Qiu, J, Feng, G, Yang, J: New results on robust energy-to-peak filtering for discrete-time switched polytopic linear systems with time-varying delay. *IET Control Theory Appl.* **2**(9), 795-806 (2008)
16. Zhou, Y, Li, J: Energy-to-peak filtering for singular systems: the discrete-time case. *IET Control Theory Appl.* **2**(9), 773-781 (2008)
17. Ahn, C: $\mathcal{L}_2 - \mathcal{L}_\infty$ nonlinear system identification via recurrent neural networks. *Nonlinear Dyn.* **62**, 543-552 (2010)
18. Ahn, C: $\mathcal{L}_2 - \mathcal{L}_\infty$ chaos synchronization. *Prog. Theor. Phys.* **123**(3), 421-430 (2010)
19. Ahn, C: An answer to the open problem of $\mathcal{L}_2 - \mathcal{L}_\infty$ synchronization for time-delayed chaotic systems. *Eur. Phys. J. Plus* **127**(2), 22 (2012)
20. Arnold, V: *Mathematical Methods of Classical Mechanics*. Springer, New York (1989)
21. Boyd, S, Ghaoui, LE, Feron, E, Balakrishnan, V: *Linear Matrix Inequalities in Systems and Control Theory*. SIAM, Philadelphia (1994)
22. Gahinet, P, Nemirovski, A, Laub, AJ, Chilali, M: *LMI Control Toolbox*. The Mathworks, Natic (1995)

doi:10.1186/1687-1847-2012-150

Cite this article as: Ahn and Lee: Sets of generalized \mathcal{H}_2 exponential stability criteria for switched multilayer dynamic neural networks. *Advances in Difference Equations* 2012 **2012**:150.