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*Research Article*

# Parameter Identification and Synchronization of Dynamical System by Introducing an Auxiliary Subsystem

**Haipeng Peng,[1,2,3] Lixiang Li,[1,2,3] Fei Sun,[1,2,3] Yixian Yang,[1,2,3] and Xiaowen Li[1]**

[1] *Information Security Center, State Key Laboratory of Networking and Switching Technology, Beijing University of Posts and Telecommunications, P.O. Box 145, Beijing 100876, China*
[2] *Key Laboratory of Network and Information Attack and Defence Technology of Ministry of Education, Beijing University of Posts and Telecommunications, Beijing 100876, China*
[3] *National Engineering Laboratory for Disaster Backup and Recovery, Beijing University of Posts and Telecommunications, Beijing 100876, China*

Correspondence should be addressed to Lixiang Li, li_lixiang2006@yahoo.com.cn

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We propose a novel approach of parameter identification using the adaptive synchronized observer by introducing an auxiliary subsystem, and some sufficient conditions are given to guarantee the convergence of synchronization and parameter identification. We also demonstrate the mean convergence of synchronization and parameters identification under the influence of noise. Furthermore, in order to suppress the influence of noise, we complement a filter in the output. Numerical simulations on Lorenz and Chen systems are presented to demonstrate the effectiveness of the proposed approach.

## 1. Introduction

Since the pioneering work of Pecora and Carroll [1], chaos synchronization has become an active research subject due to its potential applications in physics, chemical reactions, biological networks, secure communication, control theory, and so forth [2–12]. An important application of synchronization is in adaptive parameter estimation methods where parameters in a model are adjusted dynamically in order to minimize the synchronization error [13–15]. To achieve system synchronization and parameter convergence, there are two general approaches based on the typical Lyapunov's direct method [2–9] or LaSalle's principle [10]. When adaptive synchronization methods are applied to identify the uncertain parameters, some restricted conditions on dynamical systems, such as persistent excitation

(PE) condition [11, 15] or linear independence (LI) conditions [10], should be matched to guarantee that the estimated parameters converge to the true values [12].

In the following, we explore a novel method for parameter estimation by introducing an auxiliary subsystem in adaptive synchronized observer instead of Lyapunov's direct method and LaSalle's principle. It will be shown that through harnessing the auxiliary subsystem, parameters can be well estimated from a time series of dynamical systems based on adaptive synchronized observer. Moreover, noise plays an important role in parameter identification. However, little attention has been given to this point. Here we demonstrate the mean convergence of synchronization and parameters identification under the influence of noise. Furthermore, we implement a filter to recover the performance of parameter identification suppressing the influence of the noise.

## 2. Parameter Identification Method

In the master-slave framework, consider the following master system:

$$\dot{x}_i = \theta_i f_i(x) + g_i(x), \quad (i = 1, 2, \dots, n), \tag{2.1}$$

where $x = (x_1, x_2, \dots, x_n)$ is the state vector, $\theta_i$ is the unique unknown parameter to be identified, and $f_i, g_i : R^n \to R$ are the nonlinear functions of the state vector $x$ in the $i$th equation.

In order to obtain our main results, the auxiliary subsystem is needed

$$\dot{\gamma} = -L\gamma + f(x), \tag{2.2}$$

where $L$ is a positive constant.

**Lemma 2.1.** *If $f(x)$ is bounded and does not converge to zero as $t \to \infty$, then the state $\gamma$ of system* (2.2) *is bounded and does not converge to zero, when $t \to \infty$.*

*Proof.* If $f(x)$ is bounded, we can easily know that $\gamma$ is bounded [16]. We suppose that the state $\gamma$ of system (2.2) converges to zero, when $t \to \infty$. According to LaSalle principle, we have the invariant set $\gamma = 0$, then $\dot{\gamma} = 0$; therefore, from system (2.2), we get $f(x) \to 0$ as $t \to \infty$. This contradicts the condition that $f(x)$ does not converge to zero as $t \to \infty$. Therefore, the state $\gamma$ does not converge to zero, when $t \to \infty$.

Based on observer theory, the following response system is designed to synchronize the state vector and identify the unknown parameters. □

**Theorem 2.2.** *If Lemma 2.1 holds, then the following response system* (2.3) *is an adaptive synchronized observer for system* (2.1)*, in the sense that for any set of initial conditions, $y_i \to x_i$ and $\widehat{\theta}_i \to \theta_i$ as $t \to \infty$.*

$$\begin{aligned}
\dot{y}_i &= g_i(x) + f_i(x)\widehat{\theta}_i + (y_i - x_i)\left(-L_i - k_i\gamma_i^2(t)\right), \\
\dot{\widehat{\theta}}_i &= k_i\gamma_i(t)(x_i - y_i), \\
\dot{\gamma}_i(t) &= -L_i\gamma_i + f_i(x),
\end{aligned} \tag{2.3}$$

*where $y_i, \widehat{\theta}_i$ are the observed state and estimated parameter of $x_i$ and $\theta_i$, respectively, and $k_i$ and $L_i$ are positive constants.*

*Proof.* From system (2.3), we have

$$\dot{y}_i = g_i(x) + f_i(x)\widehat{\theta}_i + (y_i - x_i)(-L_i) + \gamma_i(t)\dot{\widehat{\theta}}_i. \tag{2.4}$$

Let $e_i = y_i - x_i$, $\widetilde{\theta}_i = \widehat{\theta}_i - \theta_i$, $w_i(t) = e_i(t) - \widetilde{\theta}_i\gamma_i(t)$, and note that $\dot{\theta}_i = 0$; then

$$\begin{aligned} \dot{w}_i(t) &= -L_i e_i + f_i(x)\widetilde{\theta}_i + \gamma_i(t)\dot{\widehat{\theta}}_i - \dot{\gamma}_i(t)\widetilde{\theta}_i - \gamma_i(t)\dot{\widetilde{\theta}}_i \\ &= -L_i\left(w_i(t) + \gamma_i(t)\widetilde{\theta}_i\right) + f_i(x)\widetilde{\theta}_i - \dot{\gamma}_i(t)\widetilde{\theta}_i \\ &= -L_i w_i(t) + \widetilde{\theta}_i\left(-L_i\gamma_i(t) + f_i(x) - \dot{\gamma}_i(t)\right). \end{aligned} \tag{2.5}$$

Since $\gamma_i(t)$ is generated by (2.3), then

$$\dot{w}_i(t) = -L_i w_i(t). \tag{2.6}$$

Obviously, $w_i(t) \to 0$ as $t \to \infty$.

From $\dot{\widehat{\theta}}_i = k_i\gamma_i(t)(x_i - y_i)$ and $\dot{\theta}_i = 0$, we have

$$\begin{aligned} \dot{\widetilde{\theta}}_i &= \dot{\widehat{\theta}}_i - \dot{\theta}_i \\ &= -k_i\gamma_i(t)e_i \\ &= -k_i\gamma_i(t)\left(w_i(t) + \gamma_i(t)\widetilde{\theta}_i\right). \end{aligned} \tag{2.7}$$

Let us focus on the homogeneous part of system (2.7), which is

$$\dot{\widetilde{\theta}}_i = -k_i\gamma_i^2(t)\widetilde{\theta}_i. \tag{2.8}$$

The solution of system (2.8) is $\widetilde{\theta}_i(t) = \widetilde{\theta}_i(0)e^{-\int_0^t k_i\gamma_i^2(s)ds}$. From the lemma, we know that $\gamma_i(t)$ does not converge to zero. According to Barbalat theorem, we have $\int_0^t k_i\gamma_i^2(s)ds \to \infty$ as $t \to \infty$; correspondingly, $\widetilde{\theta}_i \to 0$ as $t \to \infty$, that is, the system $\dot{\widetilde{\theta}}_i = -k_i\gamma_i^2(t)\widetilde{\theta}_i$ is asymptotically stable.

Now from the exponential convergence of $w_i(t)$ in system (2.6) and asymptotical convergence of $\widetilde{\theta}_i$ in system (2.8), we obtain that $\widetilde{\theta}_i$ in system (2.7) are asymptotical convergent to zero.

Finally, from $w_i(t) \to 0$, $\widetilde{\theta}_i(t) \to 0$, and $\gamma_i(t)$ being bounded, we conclude that $e_i = w_i + \gamma_i\widetilde{\theta} \to 0$ are global asymptotical convergence.

The proof of Theorem 2.2 is completed. □

*Note 1.* When $f_i(x) = 1$ and $\theta_i$ is the offset, in this condition no matter $x$ is in stable, periodic, or chaotic state, we could use system (2.3) to estimate and synchronize the system (2.1).

*Note 2.* When the system is in stable state, parameter estimation methods based on adaptive synchronization cannot work well [10]. For this paper, when the system is in stable state, such that $f_i(x) \to 0$ as $t \to \infty$, which leads to the lemma not being hold, so system (2.3) cannot be directly applied to identify the parameters. Here, we supplement auxiliary signal $s_i$ in drive system (2.1), such that $f_i(x)$ does not converge to zero as $t \to \infty$. Then the master system becomes

$$\dot{x}_i = \theta_i f_i(x) + g_i(x) + s_i, \tag{2.9}$$

and the corresponding slave system can be constructed as

$$\begin{aligned} \dot{y}_i &= g_i(x) + f_i(x)\widehat{\theta}_i + (y_i - x_i)\left(-L_i - k_i\gamma_i^2(t)\right) + s_i, \\ \dot{\widehat{\theta}}_i &= k_i\gamma_i(t)(x_i - y_i), \\ \dot{\gamma}_i &= -L_i\gamma_i + f_i(x). \end{aligned} \tag{2.10}$$

In doing so, synchronization of the system and parameters estimation can be achieved.

## 3. Application of the Above-Mentioned Scheme

To demonstrate and verify the performance of the proposed method, numerical simulations are presented here. We take Lorenz system as the master system [17], which is described by

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1), \\ \dot{x}_2 &= (b - x_3)x_1 - x_2, \\ \dot{x}_3 &= x_1x_2 - cx_3, \end{aligned} \tag{3.1}$$

where the parameters $a$, $b$, and $c$ are unknown, and all the states are measurable. When $a = 10$, $b = 28$, $c = 8/3$, Lorenz system is chaotic.

We construct the slave systems as follows:

$$\begin{aligned} \dot{y}_1 &= (x_2 - x_1)\widehat{a} + (y_1 - x_1)\left(-L_1 - k_1\gamma_1^2(t)\right), \\ \dot{y}_2 &= (-x_1x_3 - x_2) + x_1\widehat{b} + (y_2 - x_2)\left(-L_2 - k_2\gamma_2^2(t)\right), \\ \dot{y}_3 &= x_1x_2 - x_3\widehat{c} + (y_3 - x_3)\left(-L_3 - k_3\gamma_3^2(t)\right), \\ \dot{\widehat{a}} &= k_1\gamma_1(t)(x_1 - y_1), \\ \dot{\gamma}_1(t) &= -L_1\gamma_1 + (x_2 - x_1), \\ \dot{\widehat{b}} &= k_2\gamma_2(t)(x_2 - y_2), \\ \dot{\gamma}_2(t) &= -L_2\gamma_2 + x_1, \\ \dot{\widehat{c}} &= k_3\gamma_3(t)(x_3 - y_3), \\ \dot{\gamma}_3(t) &= -L_3\gamma_3 - x_3. \end{aligned} \tag{3.2}$$

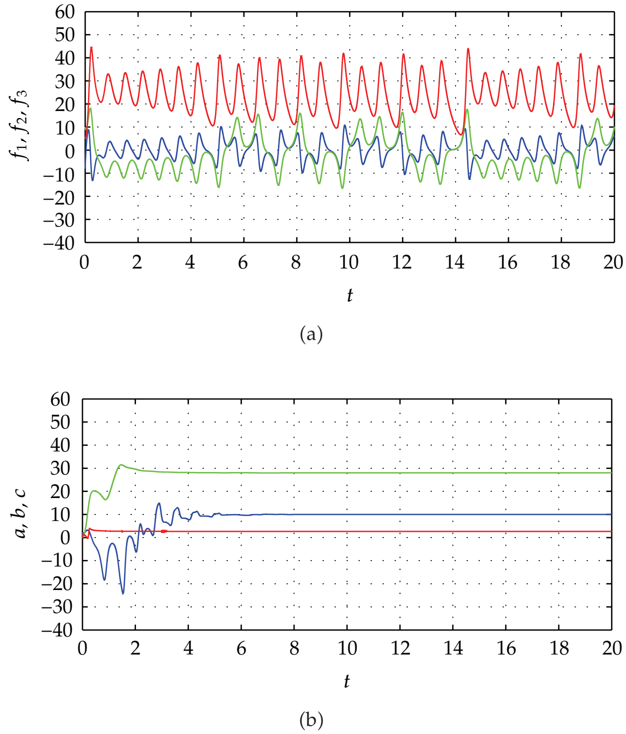



**Figure 1:** (a) The curves of $[f_1, f_2, f_3] = [(x_2 - x_1), x_1, x_3]$; (b) Identified results of $a$, $b$, $c$ versus time.

When the Lorenz system is in chaotic state, all states of $[f_1, f_2, f_3] = [(x_2 - x_1), x_1, x_3]$ are not convergent to zero as $t \to \infty$ (see Figure 1(a)). Then according to Theorem 2.2, we realize that not only the synchronization can be achieved but also the unknown parameters $a$, $b$, and $c$ can be estimated at the same time.

Figure 1(a) shows the curves of $[f_1, f_2, f_3] = [(x_2 - x_1), x_1, x_3]$. All parameters $a = 10$, $b = 28$, and $c = 8/3$ are estimated accurately and depicted in Figure 1(b). Figures 2(a)–2(c) display the results of synchronization for systems (3.1) and (3.2), where the initial conditions of simulation are $[x_1(0), x_2(0), x_3(0)] = [10, 2, 5]$, $[k_1, k_2, k_3] = [100, 1, 10]$, and $y_1(0) = y_2(0) = y_3(0) = 0, L_1 = L_2 = L_3 = 1$.

When $a = 1$, $b = 28$, and $c = 8/3$, the states of Lorenz system are not chaotic but convergent to a fixed point. Figure 3(a) shows the curves of $[f_1, f_2, f_3] = [(x_2 - x_1), x_1, x_3]$. In this case, as displayed in Figure 3(a), $f_1 = x_2 - x_1$ convergence to zero as $t \to \infty$. Figure 3(b) depicts the estimated results of parameters $a$, $b$, and $c$. From Figure 3(b), we can see that parameters $b = 28$, and $c = 8/3$ have been estimated accurately. However, the parameter $a = 1$ cannot be estimated well. According to the analysis of Note 2, we add an auxiliary signal $s = \sin(t)$ in the first subsystem of master system (3.1) and we obtain $\dot{x}_1 = a(x_2 - x_1) + \sin(t)$, such that all states of $[f_1, f_2, f_3] = [(x_2 - x_1), x_1, x_3]$ do not converge to zero as $t \to \infty$. The curves of $[(x_2 - x_1), x_1, x_3]$ are shown in Figure 4(a). Correspondingly, we add signal $s = \sin(t)$ in the first subsystem of slave system (3.2) and we have $\dot{y}_1 = (x_2 - x_1)\widehat{a} + (y_1 - x_1)(-L_1 + k_1\gamma_1^2(t)) + \sin(t)$; then all parameters $a = 1$, $b = 28$, and $c = 8/3$ are estimated accurately and depicted in Figure 4(b).

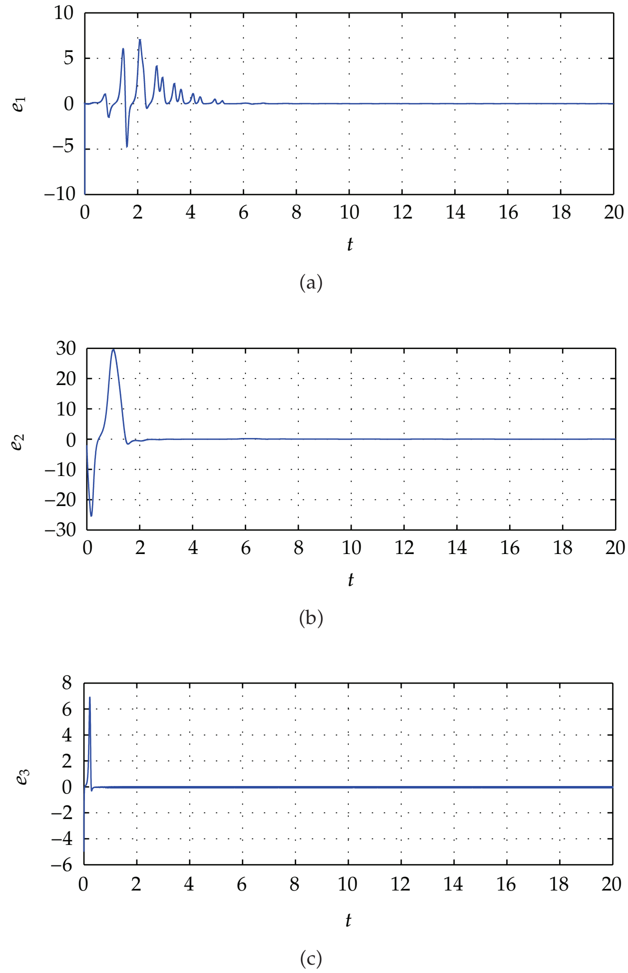
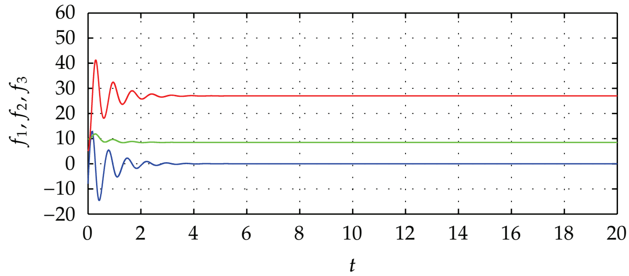



**Figure 2:** (a) The curve of $e_1$; (b) The curve of $e_2$; (c) The curve of $e_3$.

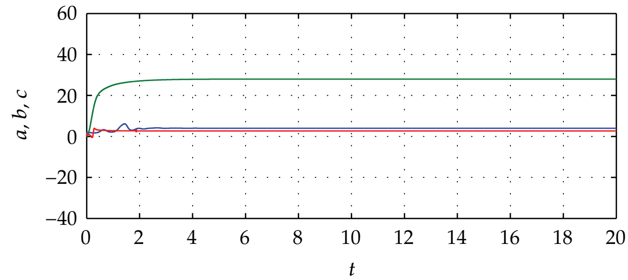
In recent years, more novel chaotic systems are found such as Chen system [18], Lü system [19], and Liu system [20]. Let us consider the identification problem for Chen system. We take Chen system as the master system, which is described by

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1), \\ \dot{x}_2 &= b(x_2 + x_1) - ax_1 - x_3x_1, \\ \dot{x}_3 &= x_1x_2 - cx_3, \end{aligned} \tag{3.3}$$

where the parameters $a$, $b$, and $c$ are unknown, and all the states are measurable. When $a = 35$, $b = 28$, and $c = 3$, Chen system is chaotic.

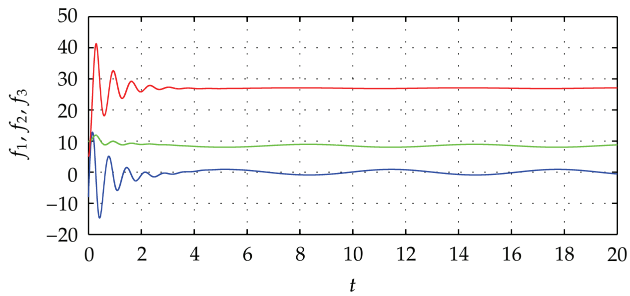



(a)

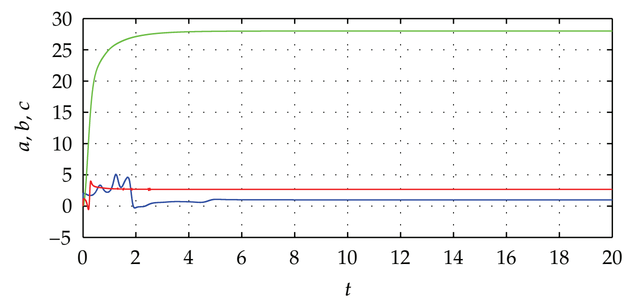



(b)

**Figure 3:** (a) The curves of $[f_1, f_2, f_3] = [(x_2 - x_1), x_1, x_3]$; (b) Identified results of $a$, $b$, $c$ versus time.




(a)




(b)

**Figure 4:** (a) The curves of $[f_1, f_2, f_3] = [(x_2 - x_1), x_1, x_3]$; (b) Identified results of $a$, $b$, $c$ versus time.

We construct the slave systems as follows:

$$\begin{aligned}
\dot{y}_1 &= (x_2 - x_1)\hat{a} + (y_1 - x_1)\left(-L_1 - k_1\gamma_1^2(t)\right),\\
\dot{y}_2 &= -x_1x_3 + \hat{b}(x_2 + x_1) - x_1\hat{a} + (y_2 - x_2)\left(-L_2 - k_2\gamma_2^2(t)\right),\\
\dot{y}_3 &= x_1x_2 - x_3\hat{c} + (y_3 - x_3)\left(-L_3 - k_3\gamma_3^2(t)\right),\\
\dot{\hat{a}} &= k_1\gamma_1(t)(x_1 - y_1),\\
\dot{\gamma}_1(t) &= -L_1\gamma_1 + (x_2 - x_1),\\
\dot{\hat{b}} &= k_2\gamma_2(t)(x_2 - y_2),\\
\dot{\gamma}_2(t) &= -L_2\gamma_2 + x_2 + x_1,\\
\dot{\hat{c}} &= k_3\gamma_3(t)(x_3 - y_3),\\
\dot{\gamma}_3(t) &= -L_3\gamma_3 - x_3.
\end{aligned} \tag{3.4}$$

Figures 5 and 6 show the synchronization error and identification results, respectively, and where $[x_1(0), x_2(0), x_3(0)] = [1, 3, 7], [k_1, k_2, k_3] = [1, 2, 3]$, and $[y_1(0), y_2(0), y_3(0)] = [0, 0, 0], [L_1, L_2, L_3] = [3, 5, 7]$.

From the simulation results of Lorenz and Chen system above, we can see that the unknown parameters could be identified. It indicates that the proposed parameter identifier in this paper could be used as an effective parameter estimator.

## 4. Parameter Identification in the Presence of Noise

Noise plays an important role in synchronization and parameters identification of dynamical systems. Noise usually deteriorates the performance of parameter identification and results in the drift of parameter identification around their true values. Here we consider the influence of noise. Suppose that there are addition noise in drive system (2.1).

$$\dot{x}_i = \theta_i f_i(x) + g_i(x) + \eta_i, \quad (i = 1, 2, \ldots, n), \tag{4.1}$$

where $\eta_i$ is the zero mean, bounded noise.

**Theorem 4.1.** *If the above lemma is hold and $\eta_i$ is independent to $f_i(x), g_i(x)$, and $\gamma_i(t)$, using the synchronized observer* (2.3)*, then for any set of initial conditions, $E(e_i)$ and $E(\widetilde{\theta}_i(t))$ converge to zero asymptotically as $t \to \infty$, where $E(e_i)$ and $E(\widetilde{\theta}_i(t))$ are mean values of $e_i$ and $\widetilde{\theta}_i(t)$, respectively.*

*Proof.* Similarly with the proof of Theorem 2.2, let $w_i = e_i - \gamma_i\widetilde{\theta}_i$; then

$$\begin{aligned}
\dot{w}_i &= -L_iw_i(t) + \widetilde{\theta}_i(-L_i\gamma_i(t) + f_i(x) - \dot{\gamma}_i) + \eta_i,\\
\dot{\widetilde{\theta}}_i &= -k_i\gamma_i(t)\left(w_i + \gamma_i(t)\widetilde{\theta}_i\right).
\end{aligned} \tag{4.2}$$

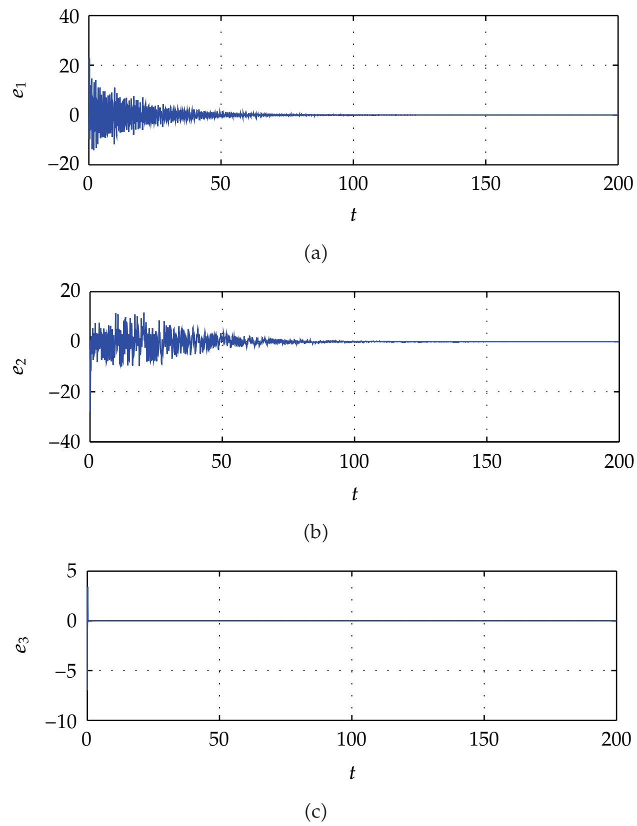



**Figure 5:** The curves of $e_1$, $e_2$, and $e_3$.

We have $\dot{w}_i = -L_i w_i(t) + \eta_i$; then

$$\frac{dE(w_i)}{dt} = -L_i E(w_i(t)) + E(\eta_i),$$
$$\frac{dE\left(\tilde{\theta}_i\right)}{dt} = E(-k_i\gamma_i(t)w_i) + E\left(-k_i\gamma_i^2\tilde{\theta}_i\right), \tag{4.3}$$

$\eta_i$ is independent to $f_i(x), g_i(x)$, and $\gamma_i(t)$, and note that $E(\eta_i) = 0$; then

$$\frac{dE(w_i)}{dt} = -L_i E(w_i(t)),$$
$$\frac{dE\left(\tilde{\theta}_i\right)}{dt} = -k_i\gamma_i(t)\left(E(w_i) + \gamma_i(t)E\left(\tilde{\theta}_i\right)\right). \tag{4.4}$$

So similarly we have $E(w_i) \to 0$, $E(\tilde{\theta}_i) \to 0$, and therefore, $E(e_i) \to 0$ as $t \to \infty$. □

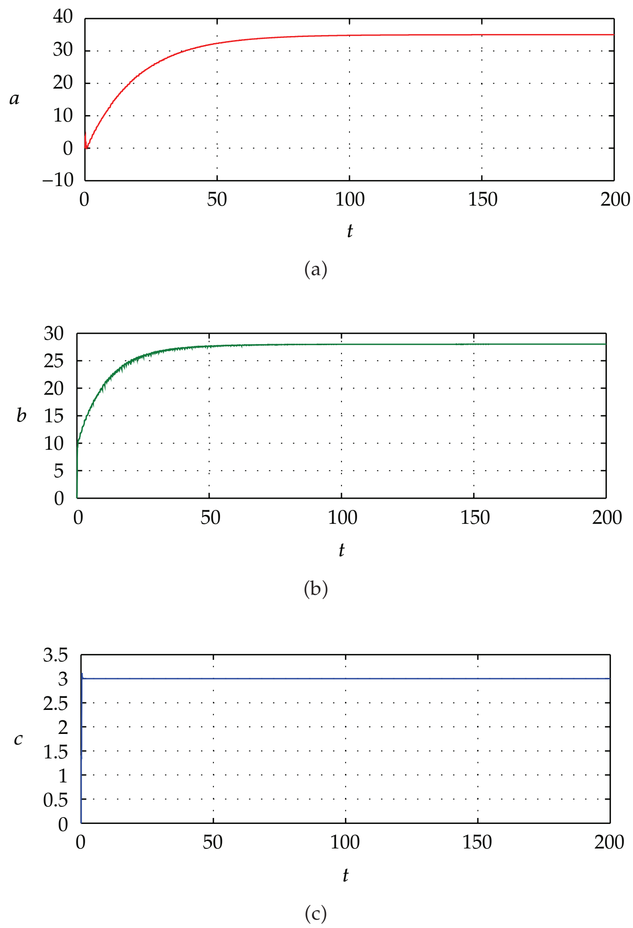



**Figure 6:** Identified results of $a$, $b$, $c$ versus time.

From Theorem 4.1, we know that that $E(\tilde{\theta}_i) \rightarrow 0$ as $t \rightarrow \infty$, which means that the estimated values for unknown parameters will fluctuate around their true values. As an illustrating example, we revisit the Lorenz system (3.1) and its slave systems (3.2), and we assume all the subsystems (3.1) are disturbed by uniformly distributed random noise with amplitude ranging from −100 to 100. Figure 7(a) shows that the estimated parameters $a$, $b$, and $c$ fluctuate around their true values.

To suppress the estimation fluctuation caused by the noise, it is suitable to use mean filters. Here we introduce the following filter:

$$\hat{\theta} = \frac{\int_0^t \hat{\theta}(s)ds}{t}. \tag{4.5}$$

It is clear to see from Figure 7(b) that unknown parameters $a$, $b$, and $c$ can be identified with high accuracy even in the presence of large random noise.

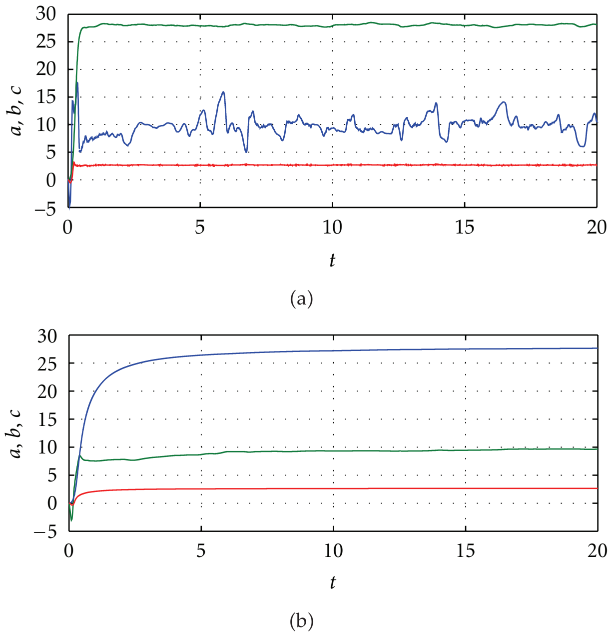



**Figure 7:** (a) Identified results of $a$, $b$, $c$ in presence of noises; (b) Identified results of $a$, $b$, $c$ in presence of noises and with filters.

## 5. Conclusions

In this paper, we propose a novel approach of identifying parameters by the adaptive synchronized observer, and a filter in the output is introduced to suppress the influence of noise. In our method, Lyapunov's direct method and LaSalle's principle are not needed. Considerable simulations on Lorenz and Chen systems are employed to verify the effectiveness and feasibility of our approach.

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