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Research Article

Impulsive Periodic Boundary Value Problems for Dynamic Equations on Time Scale

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Let $\mathbb{T}$ be a periodic time scale with period p such that $0, t_i, T = mp \in \mathbb{T}$, $i = 1, 2, \ldots, n$, $m \in \mathbb{N}$, and $0 < t_i < t_{i+1}$. Assume each t_i is dense. Using Schaeffer's theorem, we show that the impulsive dynamic equation $y^{\Delta}(t) = -a(t)y^{\sigma}(t) + f(t, y(t))$, $t \in \mathbb{T}$, $y(t_i^+) = y(t_i^-) + I(t_i, y(t_i))$, $i = 1, 2, \ldots, n$, $y(0) = y(T)$, where $y(t_i^{\pm}) = \lim_{t \to t_i^{\pm}} y(t)$, $y(t_i) = y(t_i^-)$, and y^{Δ} is the Δ-derivative on $\mathbb{T}$, has a solution.

1. Introduction

Due to their importance in numerous application, for example, physics, population dynamics, industrial robotics, optimal control, and other areas, many authors are studying dynamic equations with impulse effects; see [1–19] and references therein.

The primary motivation for this work are the papers by Kaufmann et al. [9] and Li et al. [12]. In [9], the authors used a fixed point theorem due to Krasnosel'skiĭ to establish the existence theorems for the impulsive dynamic equation:

$$\begin{gathered} y^{\Delta}(t) = -a(t)y^{\sigma}(t) + f\left(t, y(t)\right), \quad t \in (0, T] \cap \mathbb{T}, \\ y\left(t_i^+\right) = y\left(t_i^-\right) + I\left(t_i, y(t_i)\right), \quad i = 1, 2, \ldots, n, \\ y(0) = 0, \end{gathered} \tag{1.1}$$

where $y(t_i^{\pm}) = \lim_{t \to t_i^{\pm}} y(t)$, and y^{Δ} is the Δ-derivative on $\mathbb{T}$.

In [12], the authors gave sufficient conditions for the existence of solutions for the impulsive periodic boundary value problem equation:

$$\begin{gathered} u'(t) + \lambda u(t) = f(t, u(t)), \\ u(t_k^+) = u(t_k^-) + I_k(u(t_k)), \quad k = 1, 2, \dots, p, \\ u(0) = u(T), \end{gathered} \tag{1.2}$$

where $\lambda \in \mathbb{R}$, $\lambda \neq 0$, $T > 0$, and $0 = t_0 < t_1 < \cdots < t_p < t_{p+1} = T$. This paper extends and generalized the above results to dynamic equations on time scales.

We assume the reader is familiar with the notation and basic results for dynamic equations on time scales. While the books [20, 21] are indispensable resources for those who study dynamic equations on time scales, these manuscripts do not explicitly cover the concept of periodicity. The following definitions are essential in our analysis.

Definition 1.1 (see [8]). We say that a time scale $\mathbb{T}$ is *periodic* if there exist a $p > 0$ such that if $t \in \mathbb{T}$, then $t \pm p \in \mathbb{T}$. For $\mathbb{T} \neq \mathbb{R}$, the smallest positive p is called the *period* of the time scale.

Example 1.2. The following time scales are periodic:

(1) $\mathbb{T} = h\mathbb{Z}$ has period $p = h$,

(2) $\mathbb{T} = \mathbb{R}$,

(3) $\mathbb{T} = \bigcup_{k=-\infty}^{\infty} [(2k-1)h, 2kh]$, $h > 0$ has period $p = 2h$,

(4) $\mathbb{T} = \{t = k - q^m : k \in \mathbb{Z}, m \in \mathbb{N}_0\}$, where $0 < q < 1$, has period $p = 1$.

Remark 1.3. All periodic time scales are unbounded above and below.

Definition 1.4. Let $\mathbb{T} \neq \mathbb{R}$ be a periodic time scale with period p. We say that the function $f : \mathbb{T} \to \mathbb{R}$ is periodic with period T if there exists a natural number n such that $T = np$, $f(t \pm T) = f(t)$ for all $t \in \mathbb{T}$ and T is the smallest number such that $f(t \pm T) = f(t)$.

If $\mathbb{T} = \mathbb{R}$, we say that f is periodic with period $T > 0$ if T is the smallest positive number such that $f(t \pm T) = f(t)$ for all $t \in \mathbb{T}$.

Remark 1.5. If $\mathbb{T}$ is a periodic time scale with period p, then $\sigma(t \pm np) = \sigma(t) \pm np$. Consequently, the graininess function μ satisfies $\mu(t \pm np) = \sigma(t \pm np) - (t \pm np) = \sigma(t) - t = \mu(t)$ and so, is a periodic function with period p.

Let $\mathbb{T}$ be a periodic time scale with period p such that $0, t_i, T \in \mathbb{T}$, for $i = 1, 2, \dots, n$, where $T = mp$ for some $m \in \mathbb{N}$, $0 < t_i < t_{i+1}$, and assume that each t_i is dense in $\mathbb{T}$ for each $i = 1, 2, \dots, n$. We show the existence of solutions for the nonlinear periodic impulsive dynamic equation:

$$\begin{gathered} y^{\Delta}(t) = -a(t) y^{\sigma}(t) + f(t, y(t)), \quad t \in \mathbb{T}, \\ y(t_i^+) = y(t_i^-) + I(t_i, y(t_i)), \quad i = 1, 2, \dots, n, \\ y(0) = y(T), \end{gathered} \tag{1.3}$$

where $y(t_i^{\pm}) = \lim_{t \to t_i^{\pm}} y(t)$, and $y(t_i) = y(t_i^-)$. Define $[0, T] = \{t \in \mathbb{T} : 0 \leq t \leq T\}$ and note that the intervals $[a, b)$, $(a, b]$, and (a, b) are defined similarly.

In Section 2 we present some preliminary ideas that will be used in the remainder of the paper. In Section 3 we give sufficient conditions for the existence of at least one solution of the nonlinear problem (1.3).

2. Preliminaries

In this section we present some important concepts found in [20, 21] that will be used throughout the paper. We also define the space in which we seek solutions, state Schaeffer's theorem, and invert the linearized dynamic equation.

A function $p : \mathbb{T} \to \mathbb{R}$ is said to be *regressive* provided $1 + \mu(t)p(t) \neq 0$ for all $t \in \mathbb{T}^{\kappa}$. The set of all regressive rd-continuous functions $f : \mathbb{T} \to \mathbb{R}$ is denoted by $\mathcal{R}$.

Let $p \in \mathcal{R}$ and $\mu(t) \neq 0$ for all $t \in \mathbb{T}$. The *exponential function* on $\mathbb{T}$, defined by

$$e_p(t,s) = \exp\left(\int_s^t \frac{1}{\mu(z)} \operatorname{Log}\,(1 + \mu(z)p(z))\Delta z\right), \tag{2.1}$$

is the solution to the initial value problem $y^{\Delta} = p(t)y, y(s) = 1$. Other properties of the exponential function are given in the following lemma, [20, Theorem 2.36].

Lemma 2.1. *Let $p \in \mathcal{R}$. Then*

(i) $e_0(t,s) \equiv 1$ *and* $e_p(t,t) \equiv 1$;

(ii) $e_p(\sigma(t),s) = (1 + \mu(t)p(t))e_p(t,s)$;

(iii) $1/e_p(t,s) = e_{\ominus p}(t,s)$ *where,* $\ominus p(t) = -p(t)/(1 + \mu(t)p(t))$;

(iv) $e_p(t,s) = 1/e_p(s,t) = e_{\ominus p}(s,t)$;

(v) $e_p(t,s)e_p(s,r) = e_p(t,r)$;

(vi) $(1/e_p(\cdot,s))^{\Delta} = -p(t)/e_p^{\sigma}(\cdot,s)$.

Define $t_{n+1} \equiv T$ and let $J_0 = [0,t_1]$. For $i = 1,2,\ldots,n$, let $J_i = (t_i,t_{i+1}]$. Define

$$PC = \{y : \mathbb{T} \longrightarrow \mathbb{R} \mid y(t \pm T) = y(t), y \in C(J_i), y(t_i^{\pm}) \text{ exist and } y(t_i^-) = y(t_i),\ i = 1,\ldots,n\}, \tag{2.2}$$

and

$$PC^1 = \left\{y : \mathbb{T} \to \mathbb{R} \mid y(t \pm T) = y(t),\ y \in C^1(J_i),\ i = 1,\ldots,n\right\} \tag{2.3}$$

where $C(J_i)$ is the space of all real-valued continuous functions on J_i, and $C^1(J_i)$ is the space of all continuously delta-differentiable functions on J_i. The set PC is a Banach space when it is endowed with the supremum norm:

$$\|u\| = \max_{0 \leq i \leq n}\{\|u\|_i\}, \tag{2.4}$$

where $\|u\|_i = \sup_{t \in J_i}|u(t)|$.

We employ Schaeffer's fixed point theorem, see [22], to prove the existence of a periodic solution.

Theorem 2.2 (Schaeffer's Theorem). *Let S be a normed linear space and let the operator $F : S \to S$ be compact. Define*

$$H(F) = \{y \in S \mid y = \mu F(y), \mu \in (0,1)\}. \tag{2.5}$$

Then either

(i) *the set $H(F)$ is unbounded, or*

(ii) *the operator F has a fixed point in S.*

The following conditions hold throughout the paper:

(A) $a \in \mathcal{R}$ is periodic with period T; $a(t+T) = a(t)$ for all $t \in \mathbb{T}$.

(F) $f \in C(\mathbb{T} \times \mathbb{R}, \mathbb{R})$ and for all $t \in \mathbb{T}$, $f(t+T, y(t+T)) = f(t, y(t))$.

Furthermore, to ensure that the boundary value problem is not at resonance, we assume that $\eta = e_{\ominus a}(T,0) < 1$.

Consider the linear boundary value problem:

$$\begin{gathered} y^{\Delta}(t) = -a(t)y^{\sigma}(t) + \zeta(t), \quad t \in \mathbb{T}, \\ y(t_i^+) = y(t_i^-) + I(t_i, y(t_i)), \quad i = 1,2,\ldots,n, \\ y(0) = y(T), \end{gathered} \tag{2.6}$$

where $\zeta \in PC$. Our first result inverts the operator (2.6).

Lemma 2.3. *The function $y \in PC^1$ is a solution of* (2.6) *if and only if $y \in PC$ is a solution of*

$$y(t) = \int_0^T G(t,s)\zeta(s)\Delta s + \sum_{i=1}^{n} G(t,t_i)I(t_i, y(t_i)), \tag{2.7}$$

where

$$G(t,s) = \frac{1}{1-\eta}\begin{cases} e_{\ominus a}(t,s), & 0 \le s \le t \le T, \\ \eta e_{\ominus a}(t,s), & 0 \le t < s \le T. \end{cases} \tag{2.8}$$

Proof. It is easy to see that if $y \in PC^1$ is a solution of (2.6), then for $t \in [0,T]$, we have

$$y(t) = e_{\ominus a}(t,0)y(0) + \int_0^t e_{\ominus a}(t,s)\zeta(s)\Delta s + \sum_{\{i \mid t_i \le t\}} e_{\ominus a}(t,t_i)I(t_i, y(t_i)). \tag{2.9}$$

Apply the periodic boundary condition $y(0) = y(T)$ to obtain

$$y(0) = \eta y(0) + \int_0^T e_{\ominus a}(T,s)\zeta(s)\Delta s + \sum_{i=1}^{n} e_{\ominus a}(T,t_i)I\big(t_i, y(t_i)\big). \tag{2.10}$$

Since $\eta \neq 1$, we can solve the above equation for $y(0)$. Thus,

$$y(0) = \frac{1}{1-\eta}\left(\int_0^T e_{\ominus a}(T,s)\zeta(s)\Delta s + \sum_{i=1}^{n} e_{\ominus a}(T,t_i)I\big(t_i, y(t_i)\big)\right). \tag{2.11}$$

Substitute (2.11) into (2.9). Since $y \in PC^1$, we have, for all $t \in \mathbb{T}$,

$$\begin{aligned} y(t) = {} & \frac{e_{\ominus a}(t,0)}{1-\eta}\left(\int_0^T e_{\ominus a}(T,s)\zeta(s)\Delta s + \sum_{i=1}^{n} e_{\ominus a}(T,t_i)I\big(t_i, y(t_i)\big)\right) \\ & + \int_0^t e_{\ominus a}(t,s)\zeta(s)\Delta s + \sum_{\{i \mid t_i \leq t\}} e_{\ominus a}(t,t_i)I\big(t_i, y(t_i)\big). \end{aligned} \tag{2.12}$$

We can rewrite this equation as follows:

$$\begin{aligned} y(t) = {} & \frac{e_{\ominus a}(t,0)}{1-\eta}\int_t^T e_{\ominus a}(T,s)\zeta(s)\Delta s \\ & + \frac{e_{\ominus a}(t,0)}{1-\eta}\sum_{\{i \mid t_i > t\}} e_{\ominus a}(t,t_i)I\big(t_i, y(t_i)\big) \\ & + \int_0^t \left(\frac{e_{\ominus a}(t,0)e_{\ominus a}(T,s)}{1-\eta} + e_{\ominus a}(t,s)\right)\zeta(s)\Delta s \\ & + \sum_{\{i \mid t_i \leq t\}} \left(\frac{e_{\ominus a}(t,0)e_{\ominus a}(T,t_i)}{1-\eta} + e_{\ominus a}(t,t_i)\right)I\big(t_i, y(t_i)\big). \end{aligned} \tag{2.13}$$

Since $e_{\ominus a}(t,0)e_{\ominus a}(T,s) = e_{\ominus a}(T,0)e_{\ominus a}(t,s)$, then

$$\begin{aligned} y(t) = {} & \int_t^T \frac{e_{\ominus a}(T,0)e_{\ominus a}(t,s)}{1-\eta}\zeta(s)\Delta s + \int_0^t \frac{e_{\ominus a}(t,s)}{1-\eta}\zeta(s)\Delta s \\ & + \sum_{\{i \mid t_i > t\}} \frac{e_{\ominus a}(T,0)e_{\ominus a}(t,s)}{1-\eta}I\big(t_i, y(t_i)\big) \\ & + \sum_{\{i \mid t_i \leq t\}} \frac{e_{\ominus a}(t,t_i)}{1-\eta}I\big(t_i, y(t_i)\big). \end{aligned} \tag{2.14}$$

That is, y satisfies (2.7).

The converse follows trivially and the proof is complete. □

3. The Nonlinear Problem

In this section we give sufficient conditions for the existence of periodic solutions of (1.3). To this end, define the operator $N : PC \to PC$ by

$$Ny(t) = \int_0^T G(t,s)f(s,y(s))\Delta s + \sum_{i=1}^{n} G(t,t_i)I(t_i,y(t_i)). \tag{3.1}$$

Then y is a solution of (1.3) if and only if y is a fixed point of N. A standard application of the Arzelà-Ascoli theorem yields that N is compact.

Our first result is an existence and uniqueness theorem.

Theorem 3.1. *Suppose there exist constants $E_i, i = 1,\dots,n$, and L for which*

$$\left|f(t,y) - f(t,x)\right| \le L\left|y - x\right|, \quad \forall t \in \mathbb{T}, \tag{3.2}$$

and

$$\left|I(t_i,y(t_i)) - I(t_i,x(t_i))\right| \le E_i\left|y(t_i) - x(t_i)\right|, \quad i = 1,2,\dots,n, \tag{3.3}$$

and such that

$$\max_{t\in[0,T]}\left(L\int_0^T |e_{\ominus a}(t,s)|\Delta s + \sum_{i=1}^{n} E_i|e_{\ominus a}(t,t_i)|\right) < 1 - \eta. \tag{3.4}$$

Then there exists a unique solution to (1.3).

Proof. We will show that there exists a unique solution $y(t)$ of (3.1). By Lemma 2.3 this solution is the unique solution of (1.3).

Let $y, x \in PC$. Then for all $t \in \mathbb{T}$

$$\begin{aligned}\left|Ly(t) - Lx(t)\right| &\le \int_0^T |G(t,s)|\left|f(s,y(s)) - f(s,x(s))\right|\Delta s \\ &\quad + \sum_{i=1}^{n} |G(t,t_i)|\left|I(t_i,y(t_i)) - I(t_i,x(t_i))\right| \\ &\le \frac{\|y - x\|}{1-\eta}\left(L\int_0^T e_{\ominus a}(t,s)\Delta s + \sum_{i=1}^{n} E_i|e_{\ominus a}(t,t_i)|\right) \\ &< \|y - x\|.\end{aligned} \tag{3.5}$$

Hence, $\|Ly - Lx\| \le \|y - x\|$. By the Contraction Mapping Principal, there exists a unique solution of (3.1) and the proof is complete. □

Our next two results utilize Theorem 2.2 to establish the existence of solutions of (1.3).

Theorem 3.2. *Assume there exist functions* $g_1, g_2, g_3, g_4 : PC \to PC$ *with*

$$\begin{aligned} \alpha_1 &\equiv \max_{t\in[0,T]} \int_0^t |e_{\ominus a}(t,s)| g_1(s)\Delta s < \infty, \\ \beta_1 &\equiv \max_{t\in[0,T]} \int_0^t |e_{\ominus a}(t,s)| g_2(s)\Delta s < \infty, \\ \alpha_2 &\equiv \max_{t\in[0,T]} \sum_{i=1}^{n} |e_{\ominus a}(t,t_i)| g_3(t_i) < \infty, \\ \beta_2 &\equiv \max_{t\in[0,T]} \sum_{i=1}^{n} |e_{\ominus a}(t,t_i)| g_4(t_i) < \infty, \end{aligned} \tag{3.6}$$

such that

$$\begin{aligned} |f(t,y)| &\le g_1(t) + g_2(t)|y|, \quad t \in \mathbb{T}, y \in \mathbb{R}, \\ |I(t,y)| &\le g_3(t) + g_4(t)|y|, \quad t \in \mathbb{T}, y \in \mathbb{R}. \end{aligned} \tag{3.7}$$

Suppose that $\eta + \beta_1 + \beta_2 < 1$. *Then there exists at least one solution of* (1.3).

Proof. Define

$$H(N) = \{ y \in PC y(t) = \mu N y(t), \mu \in (0,1), t \in \mathbb{T} \}, \tag{3.8}$$

and let $y \in H(N)$. We show $H(N)$ is bounded by a constant that depends only on the constants $\alpha_1, \alpha_2, \beta_1, \beta_2$, and η. For all $t \in \mathbb{T}$,

$$\begin{aligned} |y(t)| &\le \mu \int_0^T |G(t,s)| |f(t,y(s))| \Delta s + \mu \sum_{i=1}^{n} |G(t,t_i)| |I(t_i, y(t_i))| \\ &\le \frac{\mu}{1-\eta} \int_0^T |e_{\ominus a}(t,s)| (g_1(s) + g_2(s)\|y\|) \Delta s \\ &\quad + \frac{\mu}{1-\eta} \sum_{i=1}^{n} |e_{\ominus a}(t,t_i)| (g_3(t_i) + g_4(t_i)\|y\|) \\ &\le \frac{1}{1-\eta} \left(\int_0^T |e_{\ominus a}(t,s)| g_1(s)\Delta s + \sum_{i=1}^{n} |e_{\ominus a}(t,t_i)| g_3(t_i) \right) \\ &\quad + \frac{\|y\|}{1-\eta} \left(\int_0^T |e_{\ominus a}(t,s)| g_2(s)\Delta s + \sum_{i=1}^{n} |e_{\ominus a}(t,t_i)| g_4(t_i) \right) \\ &\le \frac{\alpha_1 + \alpha_2}{1-\eta} + \|y\| \frac{\beta_1 + \beta_2}{1-\eta}. \end{aligned} \tag{3.9}$$

Consequently,

$$\|y\|\frac{1-\eta-\beta_1-\beta_2}{1-\eta} \le \frac{\alpha_1+\alpha_2}{1-\eta}, \tag{3.10}$$

which implies that $\|y\| \le (\alpha_1+\alpha_2)/(1-\eta-\beta_1-\beta_2)$. We have that if $y \in H(N)$, then y is bounded by the constant $(\alpha_1+\alpha_2)/(1-\eta-\beta_1-\beta_2)$. The set $H(N)$ is bounded and so by Schaeffer's theorem, the operator N has a fixed point. This fixed point is a solution of (1.3) and the proof is complete. □

In our next theorem we assume that f and I are sublinear at infinity with respect to the second variable.

Theorem 3.3. *Assume that*

(F_1) $\lim_{|y|\to\infty}(f(t,y)/y) = 0$, *uniformly, and*

(I) $\lim_{|y|\to\infty}(I(t,y)/y) = 0$, *uniformly.*

Then there exists at least one solution of the boundary value problem (1.3).

Proof. Suppose that the set

$$H(N) = \{y \in PC \mid y(t) = \mu N y(t), \mu \in (0,1), t \in \mathbb{T}\} \tag{3.11}$$

is unbounded. Then there exists sequences $\{y_k\}_{k=1}^{\infty}$ and $\{\mu_k\}_{k=1}^{\infty}$, with $\|y_k\| > k$ and $\mu_k \in (0,1)$, such that

$$y_k(t) = \mu_k\int_0^T G(t,s)f(s,y_k(s))\Delta s + \mu_k\sum_{i=1}^{n} G(t,t_i)I(t_i,y(t_i)). \tag{3.12}$$

Define $v_k(t) = y_k(t)/\|y_k\|, t \in \mathbb{T}$. Then $\|v_k\| = 1, k = 1,2,\ldots,$ and

$$v_k(t) = \mu_k\int_0^T G(t,s)\frac{f(s,y_k(s))}{\|y_k\|}\Delta s + \mu_k\sum_{i=1}^{n} G(t,t_i)\frac{I(t_i,y(t_i))}{\|y_k\|}. \tag{3.13}$$

By conditions (F_1) and (I) we have

$$\left|\frac{f(s,y_k(s))}{\|y_k\|}\right| \longrightarrow 0 \quad \text{as } k \longrightarrow \infty, \tag{3.14}$$

$$\left|\frac{I(t_i,y(t_i))}{\|y_k\|}\right| \longrightarrow 0 \quad \text{as } k \longrightarrow \infty. \tag{3.15}$$

From (3.13), (3.14), and (3.15), we have that

$$|v_k(t)| \le \int_0^T |G(t,s)|\left|\frac{f(s,y_k(s))}{\|y_k\|}\right|\Delta s + \sum_{i=1}^{n}|G(t,t_i)|\left|\frac{I(t_i,y(t_i))}{\|y_k\|}\right| \longrightarrow 0, \tag{3.16}$$

as $k \to \infty$, which contradicts $\|v_k\| = 1$ for all k. Thus the set $H(N)$ is bounded. By Theorem 2.2, the operator $N : PC \to PC$ has a fixed point. This fixed point is a solution of (1.3) and the proof is complete. □

The following corollary is an immediate consequences of Theorem 3.3

Corollary 3.4. *Assume that f and I are bounded. Then there exists at least one solution of* (1.3).

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