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Research Article

Permanence of a Discrete n-Species Schoener Competition System with Time Delays and Feedback Controls

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A discrete n-species Schoener competition system with time delays and feedback controls is proposed. By applying the comparison theorem of difference equation, sufficient conditions are obtained for the permanence of the system.

1. Introduction

In 1974, Schoener [1] proposed the following competition model:

$$\begin{aligned} \dot{x} &= r_1 x\left(\frac{I_1}{x+e_1} - r_{11}x - r_{12}y - c_1\right), \\ \dot{y} &= r_2 y\left(\frac{I_2}{y+e_2} - r_{21}x - r_{22}y - c_2\right), \end{aligned} \tag{1.1}$$

where $r_i, I_i, e_i, r_{ij}, c_i$ $(i = 1,2;\ j = 1,2)$ are all positive constants.

May [2] suggested the following set of equations to describe a pair of mutualists:

$$\begin{aligned} \dot{u} &= r_1 u\left(1 - \frac{u}{a_1 + b_1 v} - c_1 u\right), \\ \dot{v} &= r_2 v\left(1 - \frac{v}{a_2 + b_2 u} - c_2 v\right), \end{aligned} \tag{1.2}$$

where u, v are the densities of the species U, V at time t, respectively. $r_i, a_i, b_i, c_i,\ i = 1, 2$ are positive constants. He showed that system (1.2) has a globally asymptotically stable equilibrium point in the region $u > 0,\ v > 0$.

Both of the above-mentioned works are considered the continuous cases. However, many authors [3–5] have argued that the discrete time models governed by difference equations are more appropriate than the continuous ones when the populations have nonoverlapping generations. Bai et al. [6] argued that the discrete case of cooperative system is more appropriate, and they proposed the following system:

$$
\begin{aligned}
x_1(k+1) &= x_1(k)\exp\left\{ r_1(k)\left[1 - \frac{x_1(k)}{a_1(k) + b_1(k)x_2(k)} - c_1(k)x_1(k)\right]\right\},\\
x_2(k+1) &= x_2(k)\exp\left\{ r_2(k)\left[1 - \frac{x_2(k)}{a_2(k) + b_2(k)x_2(k)} - c_2(k)x_1(k)\right]\right\}.
\end{aligned}
\tag{1.3}
$$

On the other hand, as was pointed out by Huo and Li [7], ecosystem in the real world is continuously disturbed by unpredictable forces which can result in changes in the biological parameters such as survival rates. Practical interest in ecology is the question of whether or not an ecosystem can withstand those unpredictable disturbances which persist for a finite period of time. In the language of control variables, we call the disturbance functions as control variables. During the last decade, many scholars did excellent works on the feedback control ecosystems (see [8–11] and the references cited therein).

Chen [11] considered the permanence of the following nonautonomous discrete N-species cooperation system with time delays and feedback controls of the form

$$
\begin{aligned}
x_i(k+1) = x_i(k)\exp\Bigg\{ & r_i(k)\left[1 - \frac{x_i(k-\tau_{ii})}{a_i(k) + \sum_{j=1, j\neq i}^{n} b_{ij}(k)x_j(k-\tau_{ij})} - c_i(k)x_i(k-\tau_{ii})\right]\\
& -d_i(k)\mu_i(k) - e_i(k)\mu_i(k-\eta_i)\Bigg\},\\
\Delta\mu_i(k) = -\alpha_i(k)\mu_i(k) &+ \beta_i(k)x_i(k) + \gamma_i(k)x_i(k-\sigma_i),
\end{aligned}
\tag{1.4}
$$

where $x_i(k)$ $(i = 1, \dots, n)$ is the density of cooperation species X_i, $\mu_i(k)$ $(i = 1, \dots, n)$ is the control variable ([11] and the references cited therein).

Motivated by the above question, we consider the following discrete n-species Schoener competition system with time delays and feedback controls:

$$
\begin{aligned}
x_i(k+1) = x_i(k)\exp\Bigg\{ & \frac{r_i(k)}{x_i(k-\tau_i) + a_i(k)} - \sum_{j=1}^{n} b_{ij}(k)x_j(k-\tau_j) - c_i(k)\\
& -d_i(k)\mu_i(k) - e_i(k)\mu_i(k-\eta_i)\Bigg\},\\
\Delta\mu_i(k) = -\alpha_i(k)\mu_i(k) &+ \beta_i(k)x_i(k) + \gamma_i(k)x_i(k-\sigma_i),
\end{aligned}
\tag{1.5}
$$

where $x_i(k)$ $(i = 1, 2, \ldots, n)$ is the density of competitive species at kth generation; $\mu_i(k)$ is the control variable; Δ is the first-order forward difference operator $\Delta\mu_i(k) = \mu_i(k+1) - \mu_i(k)$, $i = 1, 2, \ldots, n$.

Throughout this paper, we assume the following.

(H_1) $\alpha_i(k), \beta_i(k), \gamma_i(k), a_i(k), b_{ij}(k), r_i(k), c_i(k), d_i(k), e_i(k), i = 1, 2, \ldots, n$ are all bounded nonnegative sequence such that

$$\begin{gathered}
0 < \alpha_i^l \leq \alpha_i^u < 1, \quad 0 < \beta_i^l \leq \beta_i^u, \quad 0 < \gamma_i^l \leq \gamma_i^u, \quad 0 < a_i^l \leq a_i^u, \\
0 < b_{ij}^l \leq b_{ij}^u, \quad 0 < r_i^l \leq r_i^u, \quad 0 < c_i^l \leq c_i^u, \quad 0 < d_i^l \leq d_i^u, \quad 0 < e_i^l \leq e_i^u.
\end{gathered} \tag{1.6}$$

Here, for any bounded sequence $\{a(k)\}$, $a^u = \sup_{k \in N} a(k)$, $a^l = \inf_{k \in N} a(k)$.

(H_2) τ_i, η_i, σ_i, $i = 1, \ldots, n$ are all nonnegative integers.

Let $\tau = \max\{\tau_i, \eta_i, \sigma_i, i = 1, \ldots, n\}$, we consider (1.5) together with the following initial conditions:

$$\begin{gathered}
x_i(\theta) = \varphi_i(\theta), \quad \theta \in N[-\tau, 0] = \{-\tau, -\tau+1, \ldots, 0\}, \quad \varphi_i(0) > 0, \\
\mu_i(\theta) = \phi_i(\theta), \quad \theta \in N[-\tau, 0] = \{-\tau, -\tau+1, \ldots, 0\}, \quad \phi_i(0) > 0.
\end{gathered} \tag{1.7}$$

It is not difficult to see that solutions of (1.5) and (1.7) are well defined for all $k \geq 0$ and satisfy

$$x_i(k) > 0, \quad \mu_i(k) > 0 \quad \text{for } k \in Z, \; i = 1, 2, \ldots, n. \tag{1.8}$$

The aim of this paper is, by applying the comparison theorem of difference equation, to obtain a set of sufficient conditions which guarantee the permanence of the system (1.5).

2. Permanence

In this section, we establish a permanence result for system (1.5).

Definition 2.1. System (1.5) is said to be permanent if there exist positive constants M and m such that

$$\begin{gathered}
m \leq \lim_{k \to +\infty} \inf x_i(k) \leq \lim_{k \to +\infty} \sup x_i(k) \leq M, \quad i = 1, 2, \ldots, n, \\
m \leq \lim_{k \to +\infty} \inf \mu_i(k) \leq \lim_{k \to +\infty} \sup \mu_i(k) \leq M, \quad i = 1, 2, \ldots, n
\end{gathered} \tag{2.1}$$

for any solution $x(k) = (x_1(k), \ldots, x_n(k), \mu_1(k), \ldots, \mu_n(k))$ of system (1.5).

Now, let us consider the first-order difference equation

$$y(k+1) = Ay(k) + B, \quad k = 1, 2, \ldots, \tag{2.2}$$

where A, B are positive constants. Following Lemma 2.1 is a direct corollary of Theorem 6.2 of L. Wang and M. Q. Wang [12, page 125].

Lemma 2.2. *Assuming that $|A| < 1$, for any initial value y(0), there exists a unique solution y(k) of* (2.2) *which can be expressed as follow:*

$$y(k) = A^k\left(y(0) - y^*\right) + y^*, \tag{2.3}$$

where $y^* = B/(1-A)$. Thus, for any solution $\{y(k)\}$ of system (2.2)*, one has*

$$\lim_{k\to+\infty} y(k) = y^*. \tag{2.4}$$

Following comparison theorem of difference equation is Theorem 2.1 of [12, page 241].

Lemma 2.3. *Let $k \in N_{k_0}^+ = \{k_0, k_0+1, \ldots, k_0+l, \ldots\}$, $r \geq 0$. For any fixed $k, g(k,r)$ is a nondecreasing function with respect to r, and for $k \geq k_0$, the following inequalities hold:*

$$\begin{aligned} y(k+1) &\leq g\left(k, y(k)\right), \\ u(k+1) &\geq g(k, u(k)). \end{aligned} \tag{2.5}$$

If $y(k_0) \leq u(k_0)$, then $y(k) \leq u(k)$ for all $k \geq k_0$.

Now let us consider the following single species discrete model:

$$N(k+1) = N(k)\exp\{a(k) - b(k)N(k)\}, \tag{2.6}$$

where $\{a(k)\}$ and $\{b(k)\}$ are strictly positive sequences of real numbers defined for $k \in N = \{0, 1, 2, \ldots\}$ and $0 < a^l \leq a^u$, $0 < b^l \leq b^u$. Similarly to the proof of Propositions 1 and 3 [13], we can obtain the following.

Lemma 2.4. *Any solution of system* (2.6) *with initial condition $N(0) > 0$ satisfies*

$$m \leq \lim_{k\to+\infty} \inf N(k) \leq \lim_{k\to+\infty} \sup N(k) \leq M, \tag{2.7}$$

where

$$M = \frac{1}{b^l}\exp\{a^u - 1\}, \quad m = \frac{a^l}{b^u}\exp\left\{a^l - b^u M\right\}. \tag{2.8}$$

Proposition 2.5. *Assume that (H_1) and (H_2) hold, then*

$$\begin{aligned} &\lim_{k\to+\infty} \sup x_i(k) \leq M_i, \quad i = 1, \ldots, n, \\ &\lim_{k\to+\infty} \sup \mu_i(k) \leq Q_i, \quad i = 1, \ldots, n, \end{aligned} \tag{2.9}$$

where

$$M_i = \frac{1}{b_{ii}^l \exp\left\{-\left(r_i^u \tau_i / a_i^l\right)\right\}} \exp\left\{\frac{r_i^u}{a_i^l} - 1\right\}, \quad Q_i = \frac{(\beta_i^u + \gamma_i^u) M_i}{\alpha_i^l}. \tag{2.10}$$

Proof. Let $x(k) = (x_1(k), \ldots, x_n(k),\ \mu_1(k), \ldots, \mu_n(k))$ be any positive solution of system (1.5), from the ith equation of (1.5), we have

$$x_i(k+1) \le x_i(k) \exp\left\{\frac{r_i(k)}{a_i^l}\right\}. \tag{2.11}$$

Let $x_i(k) = \exp\{N_i(k)\}$, the inequality above is equivalent to

$$N_i(k+1) - N_i(k) \le \frac{r_i(k)}{a_i^l}. \tag{2.12}$$

Summing both sides of (2.12) from $k - \tau_i$ to $k - 1$ leads to

$$\sum_{j=k-\tau_i}^{k-1} (N_i(j+1) - N_i(j)) \le \sum_{j=k-\tau_i}^{k-1} \frac{r_i(j)}{a_i^l} \le \frac{r_i^u}{a_i^l} \tau_i, \tag{2.13}$$

and so,

$$N_i(k - \tau_i) \ge N_i(k) - \frac{r_i^u \tau_i}{a_i^l}, \tag{2.14}$$

therefore,

$$x_i(k - \tau_i) \ge x_i(k) \exp\left\{-\frac{r_i^u \tau_i}{a_i^l}\right\}. \tag{2.15}$$

Substituting (2.15) to the ith equation of (1.5) leads to

$$x_i(k+1) \le x_i(k) \exp\left\{\frac{r_i(k)}{a_i^l} - b_{ii}(k) \exp\left\{-\frac{r_i^u \tau_i}{a_i^l}\right\} x_i(k)\right\}. \tag{2.16}$$

By applying Lemmas 2.3 and 2.4, it immediately follows that

$$\lim_{k \to +\infty} \sup x_i(k) \le \frac{1}{b_{ii}^l \exp\left\{-r_i^u \tau_i / a_i^l\right\}} \exp\left\{\frac{r_i^u}{a_i^l} - 1\right\} := M_i. \tag{2.17}$$

For any positive constant ε small enough, it follows from (2.17) that there exists enough large K_0 such that

$$x_i(k) \le M_i + \varepsilon, \quad i = 1, \ldots, n, \quad \forall k \ge K_0. \tag{2.18}$$

From the $n + i$th equation of the system (1.5) and (2.18), we can obtain

$$\Delta\mu_i(k) \le -\alpha_i(k)\mu_i(k) + \left(\beta_i(k) + \gamma_i(k)\right)(M_i + \varepsilon), \tag{2.19}$$

for all $k \ge K_0 + \max\{\sigma_i, i = 1, \ldots, n.\}$. And so,

$$\mu_i(k+1) \le \left(1 - \alpha_i^l\right)\mu_i(k) + \left(\beta_i^u + \gamma_i^u\right)(M_i + \varepsilon), \tag{2.20}$$

for all $k \ge K_0 + \max\{\sigma_i, i = 1, 2, \ldots, n.\}$. Noticing that $0 < 1 - \alpha_i^l < 1$ $(i = 1, 2, \ldots, n)$, by applying Lemmas 2.2 and 2.3, it follows from (2.20) that

$$\lim_{k \to +\infty} \sup \mu_i(k) \le \frac{\left(\beta_i^u + \gamma_i^u\right)(M_i + \varepsilon)}{\alpha_i^l}. \tag{2.21}$$

Setting $\varepsilon \to 0$ in the inequality above leads to

$$\lim_{k \to +\infty} \sup \mu_i(k) \le \frac{\left(\beta_i^u + \gamma_i^u\right)M_i}{\alpha_i^l} := Q_i. \tag{2.22}$$

This completes the proof of Proposition 2.5. □

Now we are in the position of stating the permanence of system (1.5).

Theorem 2.6. *Assume that* (H_1) *and* (H_2) *hold, assume further that*

$$\frac{r_i^l}{M_i + a_i^u} - \sum_{j=1, j \ne i}^{n} b_{ij}^u M_j - c_i^u - \left(d_i^u + e_i^u\right)Q_i > 0, \quad i = 1, 2, \ldots, n, \tag{2.23}$$

then system (1.5) *is permanent.*

Proof. By applying Proposition 2.5, we see that to end the proof of Theorem 2.6, it is enough to show that under the conditions of Theorem 2.6,

$$\begin{gathered}\lim_{k \to +\infty} \inf x_i(k) \ge m_i, \quad i = 1, 2, \ldots, n, \\ \lim_{k \to +\infty} \inf \mu_i(k) \ge q_i, \quad i = 1, 2, \ldots, n.\end{gathered} \tag{2.24}$$

From Proposition 2.5, for all $\varepsilon > 0$, there exists a $K_1 > 0, K_1 \in N$, for all $k > K_1$,

$$x_i(k) \leq M_i + \varepsilon; \quad \mu_i(k) \leq Q_i + \varepsilon, \quad i = 1, 2, \ldots, n. \tag{2.25}$$

From the ith equation of system (1.5) and (2.25), we have

$$x_i(k+1) \geq x_i(k) \exp\{A^{\varepsilon}(k)\}, \quad \forall\, k > K_1 + \tau, \tag{2.26}$$

where

$$A^{\varepsilon}(k) = \frac{r_i(k)}{(M_i + \varepsilon) + a_i^u} - \sum_{j=1}^{n} b_{ij}(k)\left(M_j + \varepsilon\right) - c_i(k) - (d_i(k) + e_i(k))(Q_i + \varepsilon). \tag{2.27}$$

Let $x_i(k) = \exp\{N_i(k)\}$, the inequality above is equivalent to

$$N_i(k+1) - N_i(k) \geq A^{\varepsilon}(k). \tag{2.28}$$

Summing both sides of (2.28) from $k - \tau_i$ to $k - 1$ leads to

$$\sum_{j=k-\tau_i}^{k-1} \left(N_i(j+1) - N_i(j)\right) \geq (A^{\varepsilon})^l \tau_i, \tag{2.29}$$

and so,

$$N_i(k - \tau_i) \leq N_i(k) - (A^{\varepsilon})^l \tau_i, \tag{2.30}$$

where

$$(A^{\varepsilon})^l = \frac{r_i^l}{(M_i + \varepsilon) + a_i^u} - \sum_{j=1}^{n} b_{ij}^u\left(M_j + \varepsilon\right) - c_i^u - \left(d_i^u + e_i^u\right)(Q_i + \varepsilon). \tag{2.31}$$

Therefore,

$$x_i(k - \tau_i) \leq x_i(k) \exp\left\{-(A^{\varepsilon})^l \tau_i\right\}. \tag{2.32}$$

Substituting (2.32) to the ith equation of (1.5) leads to

$$\begin{aligned}x_i(k+1) \geq x_i(k)\exp\Bigg\{ & \frac{r_i(k)}{(M_i+\varepsilon)+a_i^u} - \sum_{j=1, j\neq i}^{n} b_{ij}(k)\left(M_j+\varepsilon\right) - c_i(k) \\ & -b_{ii}(k)\exp\left\{-\left(A^{\varepsilon}\right)^l \tau_i\right\} x_i(k) - \left(d_i(k)+e_i(k)\right)\left(Q_i+\varepsilon\right)\Bigg\} \\ = x_i(k)\exp & \left\{B^{\varepsilon}(k) - b_{ii}(k)\exp\left\{-\left(A^{\varepsilon}\right)^l \tau_i\right\} x_i(k)\right\},\end{aligned} \tag{2.33}$$

for all $k > K_1 + \tau$, where

$$B^{\varepsilon}(k) = \frac{r_i(k)}{(M_i+\varepsilon)+a_i^u} - \sum_{j=1, j\neq i}^{n} b_{ij}(k)\left(M_j+\varepsilon\right) - c_i(k) - \left(d_i(k)+e_i(k)\right)\left(Q_i+\varepsilon\right). \tag{2.34}$$

Condition (2.23) shows that Lemma 2.4 could be apply to (2.33), and so, by applying Lemmas 2.3 and 2.4, it immediately follows that

$$\lim_{k\to+\infty}\inf x_i(k) \geq \frac{\left(B^{\varepsilon}\right)^l}{b_{ii}^u \exp\left\{-\left(A^{\varepsilon}\right)^l \tau_i\right\}} \exp\left\{\left(B^{\varepsilon}\right)^l - b_{ii}^u \exp\left\{-\left(A^{\varepsilon}\right)^l \tau_i\right\} M_i\right\}, \tag{2.35}$$

where

$$\left(B^{\varepsilon}\right)^l = \frac{r_i^l}{(M_i+\varepsilon)+a_i^u} - \sum_{j=1, j\neq i}^{n} b_{ij}^u\left(M_j+\varepsilon\right) - c_i^u - \left(d_i^u + e_i^u\right)\left(Q_i+\varepsilon\right). \tag{2.36}$$

Setting $\varepsilon \to 0$ in (2.35) leads to

$$\lim_{k\to+\infty}\inf x_i(k) \geq \frac{\left(B^{0}\right)^l}{b_{ii}^u \exp\left\{-\left(A^{0}\right)^l \tau_i\right\}} \exp\left\{\left(B^{0}\right)^l - b_{ii}^u \exp\left\{-\left(A^{0}\right)^l \tau_i\right\} M_i\right\}, \tag{2.37}$$

where

$$\begin{aligned}\left(A^{0}\right)^l &= \frac{r_i^l}{M_i+a_i^u} - \sum_{j=1}^{n} b_{ij}^u M_j - c_i^u - \left(d_i^u + e_i^u\right)Q_i, \\ \left(B^{0}\right)^l &= \frac{r_i^l}{M_i+a_i^u} - \sum_{j=1, j\neq i}^{n} b_{ij}^u M_j - c_i^u - \left(d_i^u + e_i^u\right)Q_i.\end{aligned} \tag{2.38}$$

For any positive constant ε small enough, it follows from (2.37) that there exists enough large K_2 such that

$$x_i(k) \geq m_i - \varepsilon, \quad i = 1, \ldots, n, \quad \forall k \geq K_2. \tag{2.39}$$

From the $n + i$th equation of the system (1.5) and (2.39), we can obtain

$$\Delta\mu_i(k) \geq -\alpha_i(k)\mu_i(k) + (\beta_i(k) + \gamma_i(k))(m_i - \varepsilon), \tag{2.40}$$

for all $k \geq K_2 + \max\{\sigma_i, i = 1, \ldots, n\}$. And so,

$$\mu_i(k+1) \geq (1 - \alpha_i^u)\mu_i(k) + \left(\beta_i^l + \gamma_i^l\right)(m_i - \varepsilon), \tag{2.41}$$

for all $k \geq K_2 + \max\{\sigma_i, i = 1, 2, \ldots, n\}$. Noticing that $0 < 1 - \alpha_i^u < 1$ $(i = 1, 2, \ldots, n)$, by applying Lemmas 2.2 and 2.3, it follows from (2.41) that

$$\lim_{k \to +\infty} \inf \mu_i(k) \geq \frac{\left(\beta_i^l + \gamma_i^l\right)(m_i - \varepsilon)}{\alpha_i^u}. \tag{2.42}$$

Setting $\varepsilon \to 0$ in the inequality above leads to

$$\lim_{k \to +\infty} \inf \mu_i(k) \geq \frac{\left(\beta_i^l + \gamma_i^l\right)m_i}{\alpha_i^u}. \tag{2.43}$$

This ends the proof of Theorem 2.6. □

Now let us consider the following discrete N-species Schoener competition system with time delays:

$$x_i(k+1) = x_i(k)\exp\left\{\frac{r_i(k)}{x_i(k - \tau_i) + a_i(k)} - \sum_{j=1}^{n} b_{ij}(k)x_j(k - \tau_j) - c_i(k)\right\}, \tag{2.44}$$

where $x_i(k)$ $(i = 1, \ldots, n)$ is the density of species X_i. Obviously, system (2.44) is the generalization of system (1.5). From the previous proof, we can immediately obtain the following theorem.

Theorem 2.7. *Assume that* (H_1) *and* (H_2) *hold, assume further that*

$$\frac{r_i^l}{M_i + a_i^u} - \sum_{j=1, j \neq i}^{n} b_{ij}^u M_j - c_i^u > 0, \quad i = 1, 2, \ldots, n, \tag{2.45}$$

then system (2.44) *is permanent.*

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