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Research Article

On a Conjecture for a Higher-Order Rational Difference Equation

Maoxin Liao,[1,2] Xianhua Tang,[1] and Changjin Xu[1,3]

[1] *School of Mathematical Sciences and Computing Technology, Central South University, Changsha, Hunan 410083, China*
[2] *School of Mathematics and Physics, University of South China, Hengyang, Hunan 421001, China*
[3] *College of Science, Hunan Institute of Engineering, Xiangtan, Hunan 411104, China*

Correspondence should be addressed to Maoxin Liao, maoxinliao@163.com

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This paper studies the global asymptotic stability for positive solutions to the higher order rational difference equation $x_n = (\prod_{j=1}^{m}(x_{n-k_j}+1)+\prod_{j=1}^{m}(x_{n-k_j}-1))/(\prod_{j=1}^{m}(x_{n-k_j}+1)-\prod_{j=1}^{m}(x_{n-k_j}-1))$, $n = 0,1,2,\ldots$, where m is odd and $x_{-k_m}, x_{-k_m+1},\ldots,x_{-1} \in (0,\infty)$. Our main result generalizes several others in the recent literature and confirms a conjecture by Berenhaut et al., 2007.

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1. Introduction

In 2007, Berenhaut et al. [1] proved that every solution of the following rational difference equation

$$x_n = \frac{x_{n-k}+x_{n-m}}{1+x_{n-k}x_{n-m}}, \quad n = 0,1,2,\ldots \tag{1.1}$$

converges to its unique equilibrium 1, where $x_{-m}, x_{-m+1},\ldots,x_{-1} \in (0,\infty)$ and $1 \le k < m$. Based on this fact, they put forward the following two conjectures.

Conjecture 1.1. *Suppose that $1 \le k < l < m$ and that $\{x_n\}$ satisfies*

$$x_n = \frac{x_{n-k}+x_{n-l}+x_{n-m}+x_{n-k}x_{n-l}x_{n-m}}{1+x_{n-k}x_{n-l}+x_{n-l}x_{n-m}+x_{n-m}x_{n-k}}, \quad n = 0,1,2,\ldots \tag{1.2}$$

with $x_{-m}, x_{-m+1},\ldots,x_{-1} \in (0,\infty)$. Then, the sequence $\{x_n\}$ converges to the unique equilibrium 1.

Conjecture 1.2. *Suppose that m is odd and $1 \le k_1 < k_2 < \cdots < k_m$, and define $S = \{1, 2, \ldots, m\}$. If $\{x_n\}$ satisfies*

$$x_n = \frac{f_1(x_{n-k_1}, x_{n-k_2}, \ldots, x_{n-k_m})}{f_2(x_{n-k_1}, x_{n-k_2}, \ldots, x_{n-k_m})}, \quad n = 0, 1, 2, \ldots \tag{1.3}$$

with $x_{-k_m}, x_{-k_m+1}, \ldots, x_{-1} \in (0, \infty)$, where

$$\begin{aligned} f_1(y_1, y_2, \ldots, y_m) &= \sum_{j \in \{1,3,\ldots,m\}} \sum_{\{t_1,t_2,\ldots,t_j\} \subset S; t_1<t_2<\cdots<t_j} y_{t_1} y_{t_2} \cdots y_{t_j}, \\ f_2(y_1, y_2, \ldots, y_m) &= 1 + \sum_{j \in \{2,4,\ldots,m-1\}} \sum_{\{t_1,t_2,\ldots,t_j\} \subset S; t_1<t_2<\cdots<t_j} y_{t_1} y_{t_2} \cdots y_{t_j}. \end{aligned} \tag{1.4}$$

Then the sequence $\{x_n\}$ converges to the unique equilibrium 1.

Motivated by [2], Berenhaut et al. started with the investigation of the following difference equation $y_n = A + (y_{n-k}/y_{n-m})^p$ for $p > 0$ (see, [3, 4]). Among others, in [3] they used a transformation method, which has turned out to be very useful in studying (1.1) and (1.2) as well as in confirming Conjecture 1.1; see [5].

Some particular cases of (1.2) had been studied previously by Li in [6, 7], by using semicycle analysis similar to that in [8]. The problem concerning periodicity of semicycles of difference equations was solved in very general settings by Berg and Stević in [9], partially motivated also by [10].

In the meantime, it turned out that the method used in [11] by Çinar et al. can be used in confirming Conjecture 1.2 (see also [12]). More precisely [11, 12] use Corollary 3 from [13] in solving similar problems. For example, Çinar et al. has shown, in an elegant way, that the main result in [14] is a consequence of Corollary 3 in [13]. With some calculations it can be also shown that Conjecture 1.2 can be confirmed in this way (see [15]).

Some other related results can be found in [16–24].

In this paper, we will prove that Conjecture 1.2 is correct by using a new method. Obviously, our results generalize the corresponding works in [1, 5–7] and other literature.

2. Preliminaries and Notations

Observe that

$$\begin{aligned} f_1(y_1, y_2, \ldots, y_m) &= \frac{1}{2}\left[\prod_{j=1}^{m}(y_j + 1) + \prod_{j=1}^{m}(y_j - 1)\right], \\ f_2(y_1, y_2, \ldots, y_m) &= \frac{1}{2}\left[\prod_{j=1}^{m}(y_j + 1) - \prod_{j=1}^{m}(y_j - 1)\right]. \end{aligned} \tag{2.1}$$

Define function G as follows:

$$G(y_1, y_2, \ldots, y_m) = \frac{\prod_{j=1}^{m}(y_j + 1) + \prod_{j=1}^{m}(y_j - 1)}{\prod_{j=1}^{m}(y_j + 1) - \prod_{j=1}^{m}(y_j - 1)}, \quad y_1, y_2, \ldots, y_m > 0. \tag{2.2}$$

Then we can rewrite (1.3) as

$$x_n = \frac{\prod_{j=1}^{m}\left(x_{n-k_j} + 1\right) + \prod_{j=1}^{m}\left(x_{n-k_j} - 1\right)}{\prod_{j=1}^{m}\left(x_{n-k_j} + 1\right) - \prod_{j=1}^{m}\left(x_{n-k_j} - 1\right)}, \quad n = 0, 1, 2, \ldots, \tag{2.3}$$

or

$$x_n = G(x_{n-k_1}, x_{n-k_2}, \ldots, x_{n-k_m}), \quad n = 0, 1, 2, \ldots, \tag{2.4}$$

where m is an odd integer and $x_{-k_m}, x_{-k_m+1}, \ldots, x_{-1} \in (0, \infty)$.

The following lemma can be obtained by simple calculations.

Lemma 2.1. *Let G be defined by* (2.2). *Then*

$$\frac{\partial G}{\partial y_i} = \frac{4\prod_{j=1, j\neq i}^{m}\left(y_j^2 - 1\right)}{\left[\prod_{j=1}^{m}(y_j + 1) - \prod_{j=1}^{m}(y_j - 1)\right]^2}\begin{cases} > 0, & \prod_{j=1, j\neq i}^{m}(y_j - 1) > 0, \\ < 0, & \prod_{j=1, j\neq i}^{m}(y_j - 1) < 0, \end{cases} \tag{2.5}$$

$i = 1, 2, \ldots, m$.

Lemma 2.2. *Assume that $0 < \alpha < 1 < \beta < +\infty$. If $\alpha \le y_1, y_2, \ldots, y_m \le \beta$, then*

$$\min\{A_1, A_3, \ldots, A_m\} \le G(y_1, y_2, \ldots, y_m) \le \max\{B_1, B_3, \ldots, B_m\}, \tag{2.6}$$

where

$$\begin{aligned} A_i &= \frac{(\alpha + 1)^i (\beta + 1)^{m-i} + (\alpha - 1)^i (\beta - 1)^{m-i}}{(\alpha + 1)^i (\beta + 1)^{m-i} - (\alpha - 1)^i (\beta - 1)^{m-i}}, \\ B_i &= \frac{(\alpha + 1)^{m-i} (\beta + 1)^{i} + (\alpha - 1)^{m-i} (\beta - 1)^{i}}{(\alpha + 1)^{m-i} (\beta + 1)^{i} - (\alpha - 1)^{m-i} (\beta - 1)^{i}}, \end{aligned} \tag{2.7}$$

$i = 1, 3, \ldots, m$.

Proof. Since $G(y_1, y_2, \ldots, y_m)$ is symmetric in $y_1, y_2, \ldots, y_m$, we can assume, without loss of generality, that $\alpha \le y_1 \le y_2 \le \cdots \le y_m \le \beta$. Then there are $m+1$ possible cases:

(1) $\alpha \le 1 \le y_1 \le y_2 \le \cdots \le y_m \le \beta$;
(2) $\alpha \le y_1 \le 1 \le y_2 \le \cdots \le y_m \le \beta$;
(3) $\alpha \le y_1 \le y_2 \le 1 \le \cdots \le y_m \le \beta$;
(4) $\alpha \le y_1 \le y_2 \le y_3 \le 1 \le \cdots \le y_m \le \beta$;
$\vdots$
(m+1) $\alpha \le y_1 \le y_2 \le \cdots \le y_m \le 1 \le \beta$.

And, for the above cases (1)–(m+1), by the monotonicity of $G(y_1, y_2, \ldots, y_m)$, in turn, we may get

(1) $1 \le G(y_1, y_2, \ldots, y_m) \le B_m$;
(2) $A_1 \le G(y_1, y_2, \ldots, y_m) \le 1$;
(3) $1 \le G(y_1, y_2, \ldots, y_m) \le B_{m-2}$;
(4) $A_3 \le G(y_1, y_2, \ldots, y_m) \le 1$;
$\vdots$
(m+1) $A_m \le G(y_1, y_2, \ldots, y_m) \le 1$.

From the above inequalities, it follows that (2.6) holds. The proof is complete. □

Lemma 2.3. *Assume that* $0 < \alpha < 1 < \beta < +\infty$ *.Then*

$$A_i = \frac{(\alpha+1)^i(\beta+1)^{m-i} + (\alpha-1)^i(\beta-1)^{m-i}}{(\alpha+1)^i(\beta+1)^{m-i} - (\alpha-1)^i(\beta-1)^{m-i}} \ge \alpha, \tag{2.8}$$

$$B_i = \frac{(\alpha+1)^{m-i}(\beta+1)^i + (\alpha-1)^{m-i}(\beta-1)^i}{(\alpha+1)^{m-i}(\beta+1)^i - (\alpha-1)^{m-i}(\beta-1)^i} \le \beta, \tag{2.9}$$

$i = 1, 3, \ldots, m$.

Proof. For $i = 1, 3, \ldots, m$, it is easy to see that

$$(\alpha-1)^{i-1}(\beta-1)^{m-i} \le (\alpha+1)^{i-1}(\beta+1)^{m-i}, \tag{2.10}$$

which yields

$$(\alpha+1)(\alpha-1)^i(\beta-1)^{m-i} \ge (\alpha-1)(\alpha+1)^i(\beta+1)^{m-i}, \tag{2.11}$$

and so

$$\alpha\left[(\alpha+1)^i(\beta+1)^{m-i} - (\alpha-1)^i(\beta-1)^{m-i}\right] \le (\alpha+1)^i(\beta+1)^{m-i} + (\alpha-1)^i(\beta-1)^{m-i}. \tag{2.12}$$

It follows that (2.8) holds. Similarly, for $i = 1, 3, \ldots, m$, it is easy to see that

$$(\alpha - 1)^{m-i}(\beta - 1)^{i-1} \leq (\alpha + 1)^{m-i}(\beta + 1)^{i-1}, \tag{2.13}$$

which yields

$$(\beta + 1)(\alpha - 1)^{m-i}(\beta - 1)^{i} \leq (\beta - 1)(\alpha + 1)^{m-i}(\beta + 1)^{i}. \tag{2.14}$$

It follows that (2.9) holds. The proof is complete. □

Lemma 2.4. *Let*

$$\begin{aligned} \alpha_{j+1} &= \min\{A_{1j}, A_{3j}, \ldots, A_{mj}\}, \\ \beta_{j+1} &= \max\{B_{1j}, B_{3j}, \ldots, B_{mj}\}, \end{aligned} \tag{2.15}$$

where

$$\begin{aligned} A_{ij} &= \frac{(\alpha_j + 1)^{i}(\beta_j + 1)^{m-i} + (\alpha_j - 1)^{i}(\beta_j - 1)^{m-i}}{(\alpha_j + 1)^{i}(\beta_j + 1)^{m-i} - (\alpha_j - 1)^{i}(\beta_j - 1)^{m-i}}, \\ B_{ij} &= \frac{(\alpha_j + 1)^{m-i}(\beta_j + 1)^{i} + (\alpha_j - 1)^{m-i}(\beta_j - 1)^{i}}{(\alpha_j + 1)^{m-i}(\beta_j + 1)^{i} - (\alpha_j - 1)^{m-i}(\beta_j - 1)^{i}}, \end{aligned} \tag{2.16}$$

$i = 1, 3, \ldots, m;\ j = 0, 1, 2, \ldots.$ *Assume that* $0 < \alpha_0 < 1 < \beta_0 < +\infty$. *Then*

$$\lim_{j \to \infty} \alpha_j = \lim_{j \to \infty} \beta_j = 1. \tag{2.17}$$

Proof. By induction, we easily show that

$$0 < \alpha_j < 1 < \beta_j < +\infty, \quad j = 0, 1, 2, \ldots. \tag{2.18}$$

It follows from Lemma 2.3 that

$$\begin{aligned} A_{ij} &= \frac{(\alpha_j + 1)^{i}(\beta_j + 1)^{m-i} + (\alpha_j - 1)^{i}(\beta_j - 1)^{m-i}}{(\alpha_j + 1)^{i}(\beta_j + 1)^{m-i} - (\alpha_j - 1)^{i}(\beta_j - 1)^{m-i}} \geq \alpha_j, \\ B_{ij} &= \frac{(\alpha_j + 1)^{m-i}(\beta_j + 1)^{i} + (\alpha_j - 1)^{m-i}(\beta_j - 1)^{i}}{(\alpha_j + 1)^{m-i}(\beta_j + 1)^{i} - (\alpha_j - 1)^{m-i}(\beta_j - 1)^{i}} \leq \beta_j, \end{aligned} \tag{2.19}$$

$i = 1, 3, \ldots, m;\ j = 0, 1, 2, \ldots.$ Hence, by (2.15) and (2.18), we have

$$\alpha_j \leq \alpha_{j+1} < 1 < \beta_{j+1} \leq \beta_j, \quad j = 0, 1, 2, \ldots. \tag{2.20}$$

Equation (2.20) implies that the limits $\lim_{j\to\infty}\alpha_j$ and $\lim_{j\to\infty}\beta_j$ exist, and

$$\alpha^* = \lim_{j\to\infty}\alpha_j \in [\alpha_0, 1], \qquad \beta^* = \lim_{j\to\infty}\beta_j \in [1, \beta_0]. \tag{2.21}$$

It follows from (2.16) that

$$\begin{aligned} A_i^* &:= \lim_{j\to\infty} A_{ij} = \frac{(\alpha^*+1)^i(\beta^*+1)^{m-i} + (\alpha^*-1)^i(\beta^*-1)^{m-i}}{(\alpha^*+1)^i(\beta^*+1)^{m-i} - (\alpha^*-1)^i(\beta^*-1)^{m-i}}, \\ B_i^* &:= \lim_{j\to\infty} B_{ij} = \frac{(\alpha^*+1)^{m-i}(\beta^*+1)^i + (\alpha^*-1)^{m-i}(\beta^*-1)^i}{(\alpha^*+1)^{m-i}(\beta^*+1)^i - (\alpha^*-1)^{m-i}(\beta^*-1)^i}, \end{aligned} \tag{2.22}$$

$i = 1, 3, \ldots, m$. Let $j \to \infty$ in (2.15), we have

$$\begin{aligned} \alpha^* &= \min\{A_1^*, A_3^*, \ldots, A_m^*\}, \\ \beta^* &= \max\{B_1^*, B_3^*, \ldots, B_m^*\}. \end{aligned} \tag{2.23}$$

It follows that there exist $i, j \in \{1, 3, \ldots, m\}$ such that

$$\begin{aligned} \alpha^* &= \frac{(\alpha^*+1)^i(\beta^*+1)^{m-i} + (\alpha^*-1)^i(\beta^*-1)^{m-i}}{(\alpha^*+1)^i(\beta^*+1)^{m-i} - (\alpha^*+1)^i(\beta^*+1)^{m-i}}, \\ \beta^* &= \frac{(\alpha^*+1)^{m-j}(\beta^*+1)^j + (\alpha^*-1)^{m-j}(\beta^*-1)^j}{(\alpha^*+1)^{m-j}(\beta^*+1)^j - (\alpha^*-1)^{m-j}(\beta^*-1)^j}. \end{aligned} \tag{2.24}$$

From (2.24), we have

$$\begin{aligned} (\alpha^*-1)\left[(\alpha^*+1)^{i-1}(\beta^*+1)^{m-i} - (\alpha^*-1)^{i-1}(\beta^*-1)^{m-i}\right] &= 0, \\ (\beta^*-1)\left[(\alpha^*+1)^{m-j}(\beta^*+1)^{j-1} - (\alpha^*-1)^{m-j}(\beta^*-1)^{j-1}\right] &= 0. \end{aligned} \tag{2.25}$$

Since

$$\begin{aligned} (\alpha^*+1)^{i-1}(\beta^*+1)^{m-i} - (\alpha^*-1)^{i-1}(\beta^*-1)^{m-i} &> 0, \\ (\alpha^*+1)^{m-j}(\beta^*+1)^{j-1} - (\alpha^*-1)^{m-j}(\beta^*-1)^{j-1} &> 0, \end{aligned} \tag{2.26}$$

it follows from (2.25) and (2.18) that $\alpha^* = \beta^* = 1$. The proof is complete. □

3. Proof of Conjecture 1.2

Theorem 3.1. *Suppose that* $0 < \alpha < 1 < \beta < +\infty$ *and that*

$$x_{-k_m}, x_{-k_m+1}, \dots, x_{-1} \in [\alpha, \beta]. \tag{3.1}$$

Then the solution $\{x_n\}$ *of* (1.3) *satisfies*

$$x_n \in [\alpha, \beta], \quad for\ n = 0, 1, 2, \dots. \tag{3.2}$$

Theorem 3.1 is a direct corollary of Lemmas 2.2 and 2.3.

Proof of Conjecture 1.2. Let $\{x_n\}$ be a solution of (1.3) with $x_{-k_m}, x_{-k_m+1}, \dots, x_{-1} \in (0, \infty)$. We need to prove that

$$\lim_{n \to \infty} x_n = 1. \tag{3.3}$$

Choose $\alpha_0 \in (0, 1)$ and $\beta_0 \in (1, +\infty)$ such that

$$x_{-k_m}, x_{-k_m+1}, \dots, x_{-1} \in [\alpha_0, \beta_0]. \tag{3.4}$$

In view of Theorem 3.1, we have

$$x_n \in [\alpha_0, \beta_0], \quad n = -k_m, -k_m + 1, -k_m + 2, \dots. \tag{3.5}$$

Let α_j, β_j, A_{ij}, and B_{ij} be defined as in Lemma 2.4. Then by (3.5) and Lemma 2.2, we have

$$\begin{aligned} \min\{A_{10}, A_{30}, \dots, A_{m0}\} &\le G(x_{n-k_1}, x_{n-k_2}, \dots, x_{n-k_m}) \\ &\le \max\{B_{10}, B_{30}, \dots, B_{m0}\}, \quad n = 0, 1, 2, \dots. \end{aligned} \tag{3.6}$$

That is

$$x_n \in [\alpha_1, \beta_1], \quad n = 0, 1, 2, \dots. \tag{3.7}$$

By (3.7) and Lemma 2.2, we obtain

$$\begin{aligned} \min\{A_{11}, A_{31}, \dots, A_{m1}\} &\le G(x_{n-k_1}, x_{n-k_2}, \dots, x_{n-k_m}) \\ &\le \max\{B_{11}, B_{31}, \dots, B_{m1}\}, \quad n = k_m, k_m + 1, k_m + 2, \dots. \end{aligned} \tag{3.8}$$

That is

$$x_n \in [\alpha_2, \beta_2], \quad n = k_m, k_m + 1, k_m + 2, \dots. \tag{3.9}$$

Repeating the above procedure, in general, we can obtain

$$x_n \in [\alpha_{j+1}, \beta_{j+1}], \quad n = jk_m, jk_m + 1, jk_m + 2, \ldots, \; j = 0, 1, 2, \ldots. \tag{3.10}$$

By Lemma 2.4, we have

$$\lim_{n\to\infty} x_n = \lim_{j\to\infty} \alpha_{j+1} = \lim_{j\to\infty} \beta_{j+1} = 1, \tag{3.11}$$

which implies that (3.3) holds. The proof of Conjecture 1.2 is complete. □

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