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On pairs of fuzzy dominated mappings and applications

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Abstract

The main purpose of this paper is to present some fixed-point results for a pair of fuzzy dominated mappings which are generalized V-contractions in modular-like metric spaces. Some theorems using a partial order are discussed and also some useful results to graphic contractions for fuzzy-graph dominated mappings are developed. To explain the validity of our results, 2D and 3D graphs have been constructed. Also, applications are provided to show the novelty of our obtained results and their usage in engineering and computer science.

MSC: Pair of fuzzy mappings; Modular-like metric space; Graphic contraction; Electric circuit equations; Fractional differential equations

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1 Introduction and preliminaries

The fixed-point theory becomes essential in analysis (see [1–66]). In [19], Chistyakov firstly introduced the notion of a modular metric and discussed thoroughly its convergence, convexity, relation with metrics, convex cones, and the structure of semigroups on such spaces. The modular metric spaces generalize classical modulars over linear spaces, like Orlicz, Lebesgue, Musielak–Orlicz, Lorentz, Calderon–Lozanovskii, and Orlicz–Lorentz spaces. The main idea behind this new concept is the physical interpretation of the modular. We look at these spaces as the nonlinear version of the classical modular spaces. Padcharoen [42] initiated the idea of rational type F-contractions in modular metric spaces and proved some important results. Additional results in such spaces proved by different authors can be seen in [18, 31, 33, 37]. Nadler [39] presented a fixed-point theorem for multivalued mappings and generalized its analogues for single-valued mappings. Fixed-point results of multivalued mappings have several applications in engineering, control theory, differential equations, games and economics; see [11, 16]. In this paper, we are using multivalued mappings. Wardowski [66] introduced a new type of contractions, named F-contractions, to obtain a fixed-point result. For more results in this direction, see [2, 3, 6, 8, 15, 32, 33, 38, 55]. Here, we have used a weak family of mappings instead of the function F introduced by Wardowski. In [9] the authors observed that there are mappings which possess fixed points. Namely, they introduced a condition on closed balls to achieve common fixed points for such mappings. For further results on closed balls, see [50, 51, 63]. In this paper, we are using a sequence instead of a closed ball. Ran and Reurings [49] and Nieto and Rodríguez-López [41] gave fixed-point theory results in partially ordered sets. For more results in ordered spaces, see [20, 21, 23]. Asl et al. [10] gave the notion of α_*-admissible mappings and α–ω-contractive set-valued mappings (see also [5, 26, 29, 56]) and generalized the restriction of order. Rasham et al. [53] introduced the concept of α_*-dominated mappings to establish a new condition of order and obtained some results (see also [52, 54, 59, 62]). They proved that there are mappings which are α_*-dominated, but not α_*-admissible. The notion of fuzzy sets is introduced by Zadeh [67] and then a lot of researchers did their research work in this field. Weiss [68] and Butnariu [17] firstly discussed the concept of fuzzy mappings and showed many related results. Heilpern [16] discussed a result on fuzzy mappings, which was a further generalization of Nadler's set-valued result [39] using a Hausdorff metric. Due to importance of the Heilpern's results, fixed-point theory for fuzzy contractions using a Hausdorff metric has become more important, see [44–48, 51, 61, 62]. In this article, we prove fixed point results for a pair of fuzzy dominated maps which are generalized V-contractions and provide related graphs for 2D and 3D. An application for the solution of electric circuit equations is also presented. Moreover, a fractional differential equation is solved. Our obtained results generalize those presented in [54, 57, 59, 61, 66].

We start with the following statements which are helpful to prove our results.

Definition 1.1 ([56]) Let A be a nonempty set. A function $u : (0,1) \times A \times A \to [0,1)$ is called a modular-like metric on A if for all $a,b,c \in A; l,n > 0$, and $u_l(a,b) = u(l,a,b)$, the following hold:

(i) $u_l(a,b) = u_l(b,a)$;

(ii) $u_l(a,b) = 0$, then $a = b$;

(iii) $u_{l+n}(a,b) \leq u_l(a,c) + u_n(c,b)$.

(A,u) is called a modular-like metric space. If we replace (ii) by $u_l(a,b) = 0$ if and only if $a = b$, then (A,u) becomes a modular metric space. If we replace (ii) by $u_l(a,b) = 0$ for some $l > 0$ then $a = b$, then $(A;u)$ becomes a regular modular-like metric on A. For $e \in A$ and $\epsilon > 0, B_{u_l}(e,\epsilon) = \{p \in A : |u_l(e,p) - u_l(e,e)| \leq \epsilon)\}$ is the closed ball. We abbreviate by "*m.l.m.* space" a modular-like metric space.

Definition 1.2 ([56]) Let (A,u) be an *m.l.m.* space.

(i) A sequence $(a_n)_{n\in\mathbb{N}}$ in A is u-Cauchy for some $l > 0$, if and only if $\lim_{n,m\to+\infty} u_l(a_m,a_n)$ exists and is finite.

(ii) A sequence $(a_n)_{n\in\mathbb{N}}$ in A u-converges to $a \in A$ for some $l > 0$, if and only if $\lim_{n\to+\infty} u_l(a_n,a) = u_l(a,a)$.

(iii) $E \subseteq A$ is called u-complete if any u-Cauchy sequence $\{a_n\}$ in E is u-convergent to some $a \in E$, so that for some $l > 0$,

$$\lim_{n\to+\infty} u_l(a_n,a) = u_l(a,a) = \lim_{n,m\to+\infty} u_l(a_m,a_n).$$

Definition 1.3 ([57]) Let (A,u) be an *m.l.m.* space and $E \subseteq A$. An element p_0 of A is the closest to E if it provides the finest estimate in E for $e \in A$, i.e.,

$$u_l(e,E) = \inf_{p\in E} u_l(e,p) = u_l(e,p_0).$$

If every $e \in A$ has a greatest estimate in E, then E is identified as a proximal set. For example, let $A = \mathbb{R}^+ \cup \{0\}$ and $u_l(e,p) = \frac{1}{l}(e+p)$ for all $l > 0$. Define a set $E = [4,6]$, then for each $y \in A$,

$$u_l(y,E) = u_l\bigl(y,[4,6]\bigr) = \inf_{n\in[4,6]} u_l(y,n) = u_l(y,4).$$

Hence, 4 is the finest estimate in E for very $y \in A$. Also, $[4,6]$ is a proximal set.

From now on, denote by $P(A)$ the set of compact proximinal subsets in A.

Definition 1.4 ([56]) Let (A,u) be an *m.l.m.* space. The function ${}_{Hu_l} : P(A) \times P(A) \to [0,\infty)$, given as

$$H_{u_l}(N,M) = \max\Bigl\{\sup_{n\in N} u_l(n,M), \sup_{m\in M} u_l(N,m)\Bigr\},$$

is u_l-Hausdorff metric like. The pair $(P(A), H_{u_l})$ is named as a u_l-Hausdorff metric like space.

For examples, take $A = R^+ \cup \{0\}$. Let

$$u_l(e,p) = \frac{1}{l}(e+p) \quad \text{for all } l > 0.$$

If $N = [3,5], R = [7,8]$, then $H_{u_l}(N,R) = \frac{13}{l}$.

Definition 1.5 ([56]) Let (A,u) be an *m.l.m.* space. We will say that u satisfies the Δ_M-condition if $\lim_{n,m\to\infty} u_p(e_n,e_m) = 0$, where $p \in \mathbb{N}$ implies $\lim_{n,m\to\infty} u_l(e_n,e_m) = 0$, for some $l > 0$.

Definition 1.6 ([62]) Let A be a nonempty set, $G : A \to P(A)$, $B \subseteq A$, and $\alpha : A \times A \to [0,+\infty)$. Then G is said to be α_*-admissible on B if

$$\alpha_*(Gp,Gc) = \inf\bigl\{\alpha(u,v) : u \in Gp, v \in Gc\bigr\} \ge 1,$$

whenever $\alpha(p,c) \ge 1$, for all $p,c \in B$.

Definition 1.7 ([53]) Let A be a nonempty set, $G : A \longrightarrow P(A)$, $M \subseteq A$, and $\alpha : A \times A \to [0,+\infty)$. Then G is named as α_*-dominated on M, if for any $b \in M$,

$$\alpha_*(b,Gb) = \inf\bigl\{\alpha(b,w) : w \in Gb\bigr\} \ge 1.$$

Example 1.8 ([53]) Let $B = (-\infty,\infty)$. Define $\gamma : B \times B \to [0,\infty)$ and $K, L : B \to P(B)$, respectively, by

$$\gamma(e,r) = \begin{Bmatrix} 1 & \text{if } e > r, \\ \frac{1}{4} & \text{if } e \not> r, \end{Bmatrix}$$

and

$$Ku = [-4+u, -3+u] \quad \text{and} \quad Lr = [-2+r, -1+r].$$

Then K and L are not γ_*-admissible, but they are γ_*-dominated.

Definition 1.9 ([66]) Consider a metric space (M,d). A mapping $G:M\to M$ is called an F-contraction if for all $b,c\in M$, $\exists\tau>0$ such that $d(Gb,Gc)>0$ we have

$$\tau+F\bigl(d(Gb,Gc)\bigr)\le F\bigl(d(b,c)\bigr),$$

where $F:\mathbb{R}_+\to\mathbb{R}$ is such that:

(F1) There is $k\in(0,1)$ such that $\lim_{\sigma\to 0^+}\sigma^k F(\sigma)=0$;

(F2) F is strictly increasing;

(F3) $\lim_{n\to+\infty}\sigma_n=0$ if $\lim_{n\to+\infty}F(\sigma_n)=-\infty$, for each sequence $\{\sigma_n\}_{n=1}^{\infty}$ of positive numbers.

The family of functions verifying (F1)–(F3) is denoted by R.

Lemma 1.10 *Let (Q,u) be an m.l.m. space. Let $(P(Q),H_{u_1})$ be a u_1-Hausdorff metric like space. Then, for any $e\in C$ and for all $C,D\in P(Q)$, there is $y_e\in D$ such that*

$$H_{u_1}(C,D)\ge u_1(e,y_e).$$

Definition 1.11 ([60]) A fuzzy set U is a function from G to $[0,1]$ and $F(G)$ is the family of all fuzzy sets in G. If U is a fuzzy set and $e\in G$, then $U(e)$ is said to be the grade of membership of e in U. The β-level set of the fuzzy set U is denoted by $[U]_\beta$, and is given as

$$[U]_\beta=\bigl\{e:U(e)\ge\beta\bigr\}\quad\text{where } 0<\beta\le 1,$$
$$[U]_0=\overline{\bigl\{e:U(e)>0\bigr\}}.$$

Now, we select a subset of the family $F(G)$ of all fuzzy sets, which is a subfamily with stronger properties, i.e., the subfamily of the approximate quantities, denoted by $W(G)$.

Definition 1.12 ([24]) A fuzzy subset U of G is an approximate quantity iff its β-level set is a compact convex subset of G for each $\beta\in[0,1]$ and $\sup_{e\in G}U(e)=1$.

Definition 1.13 ([24]) Let R be an arbitrary set and G be any metric space. A fuzzy map is a mapping from R to $W(G)$. We can view a fuzzy mapping $T:R\to W(G)$ as a fuzzy subset of $R\times G$, $T:R\times G\to[0,1]$ in the sense that $T(c,y)=T(c)(y)$.

Definition 1.14 ([60]) A point $c\in M$ is called a fuzzy fixed point of a fuzzy mapping $T:M\to W(M)$ if there exists $0<\beta\le 1$ such that $c\in[Tc]_\beta$.

Definition 1.15 Let A be a nonempty set, $\xi:A\to W(A)$ be a fuzzy mapping, $M\subseteq A$, and $\alpha:A\times A\to[0,+\infty)$. Then ξ is named as fuzzy α_*-dominated on M, if for each $a\in M$ and $0<\beta\le 1$,

$$\alpha_*\bigl(a,[\xi a]_\beta\bigr)=\inf\bigl\{\alpha(a,l):l\in[\xi a]_\beta\bigr\}\ge 1.$$

Now, we are ready to prove our main theorems for a pair of fuzzy mappings which are a generalized rational type contraction.

2 Main results

Let $(\mathcal{L}, u)$ be an $m.l.m.$ space, $x_0 \in \mathcal{L}$ and $S, T : \mathcal{L} \to W(\mathcal{L})$ be fuzzy mappings on $\mathcal{L}$. Moreover, let $\gamma, \beta : \mathcal{L} \to [0;1]$ be two real functions. Let $x_1 \in S[x_0]_{\gamma(x_0)}$ be an element such that $u_1(x_0, [Sx_0]_{\gamma(x_0)}) = u_1(x_0, x_1)$. Let $x_2 \in [Tx_1]_{\beta(x_1)}$ be such that $u_1(x_1, [Tx_1]_{\beta(x_1)}) = u_1(x_1, x_2)$. Let $x_3 \in [Sx_2]_{\gamma(x_2)}$ be such that $u_1(x_2, [Sx_2]_{\gamma(x_2)}) = u_1(x_2, x_3)$. Continuing this process, we construct a sequence $\{x_n\}$ of points in $\mathcal{L}$ such that

$$x_{2n+1} \in [Sx_{2n}]_{\gamma(x_{2n})} \quad \text{and} \quad x_{2n+2} \in [Tx_{2n+1}]_{\beta(x_{2n+1})}, \quad \text{for } n = 0, 1, 2, \ldots$$

Also,

$$u_1\bigl(x_{2n}, [Sx_{2n}]_{\gamma(2n)}\bigr) = u_1(x_{2n}, x_{2n+1}), \qquad u_1\bigl(x_{2n+1}, [Tx_{2n+1}]_{\beta(x_{2n+1})}\bigr) = u_1(x_{2n+1}, x_{2n+2}).$$

We use $\{TS(x_n)\}$ to denote this sequence. We say that $\{TS(x_n)\}$ is a sequence in $\mathcal{L}$ generated by x_0.

Definition 2.1 Let $(\mathcal{L}, u)$ be a complete $m.l.m.$ space. Suppose that u is regular and the Δ_M-condition holds. Let $x_0 \in \mathcal{L}, \alpha : \mathcal{L} \times \mathcal{L} \to [0, \infty)$ and $S, T : \mathcal{L} \to W(\mathcal{L})$ be two fuzzy α_*-dominated mappings on $\{TS(x_n)\}$. The pair (S, T) is called a rational fuzzy dominated V-contraction if there exist $\tau > 0, \gamma(x), \beta(x) \in (0, 1]$ and $V \in R$ such that

$$\begin{aligned} &\tau + V\bigl(H_{u_1}\bigl([Sx]_{\gamma(x)}, [Tg]_{\beta(g)}\bigr)\bigr) \\ &\quad \leq V\left(\max\left\{u_1(x, g), u_1(x, Sx), \frac{u_2(x, [Tg]_{\beta(g)})}{2}, \frac{u_1^2(x, [Sx]_{\gamma(x)}).u_1(g, [Tg]_{\beta(g)})}{1 + u_1^2(x, g)}\right\}\right) \end{aligned} \tag{2.1}$$

whenever, $x, g \in \{TS(x_n)\}, \alpha(x, g) \geq 1,$ and $H_{u_1}([Sx]_{\gamma(x)}, [Tg]_{\beta(g)}) > 0$.

Theorem 2.2 *Let $(\mathcal{L}, u)$ be a complete m.l.m. space. Assume that $S, T : \mathcal{L} \to W(\mathcal{L})$ are two fuzzy α_*-dominated mappings on $\{TS(x_n)\}$. If (S, T) is a rational fuzzy dominated V-contraction, then $\{TS(x_n)\}$ is a Cauchy sequence in $\mathcal{L}$ and $\{TS(x_n)\} \to k \in \mathcal{L}$.*

Proof As $S, T : \mathcal{L} \to W(\mathcal{L})$ are two fuzzy α_*-dominated mappings on $\{TS(x_n)\}$, so, by definition, we have

$$\bigl(\alpha_*\{x_{2i}, [Sx_{2i}]_{\gamma(x_{2i})}\bigr) \geq 1 \quad \text{and} \quad \bigl(\alpha_*\bigl(x_{2i+1}, [Tx_{2i+1}]_{\beta(x_{2i+1})}\bigr) \geq 1$$

for all $i \in \mathbb{N}$. As $\alpha_*(x_{2i}, [Sx_{2i}]_{\gamma(x_{2i})}) \geq 1$, this implies that $\inf\{\alpha(x_{2i}, b) : b \in [Sx_{2i}]_{\gamma(x_{2i})}\} \geq 1$ and therefore, $\alpha(x_{2i}, x_{2i+1}) \geq 1$. Now, by using Lemma 1.10, we have

$$\begin{aligned} &\tau + V\bigl(u_1(x_{2i+1}, x_{2i+2})\bigr) \\ &\quad \leq \tau + V\bigl(H_{u_1}\bigl([Sx_{2i}]_{\gamma(x_{2i})}\bigr), [Tx_{2i+1}]_{\beta(x_{2i+1})}\bigr)\bigr) \\ &\quad \leq V\left(\max\left\{\begin{array}{c} u_1(x_{2i}, x_{2i+1}), u_1(x_{2i}, [Sx_{2i}]_{\gamma(x_{2i})}), \frac{u_2(x_{2i}, [Tx_{2i+1}]_{\beta(x_{2i+1})})}{2}, \\ \frac{u_1^2(x_{2i}, [Sx_{2i}]_{\gamma(x_{2i})}).u_1(x_{2i+1}, [Tx_{2i+1}]_{\beta(x_{2i+1})})}{1+u_1^2(x_{2i}, x_{2i+1})} \end{array}\right\}\right) \\ &\quad \leq V\left(\max\left\{\begin{array}{c} u_1(x_{2i}, x_{2i+1}), u_1(x_{2i}, x_{2i+1}), \frac{u_1(x_{2i}, x_{2i+1}) + u_1(x_{2i+1}, x_{2i+2})}{2}, \\ \frac{u_1^2(x_{2i}, x_{2i+1}).u_1(x_{2i+1}, x_{2i+2})}{1+u_1^2(x_{2i}, x_{2i+1})} \end{array}\right\}\right) \end{aligned}$$

$$\leq V\left(\max\left\{u_1(x_{2i},x_{2i+1}),u_1(x_{2i+1},x_{2i+2})\right\}\right).$$

This implies that

$$\tau + V\left(u_1(x_{2i+1},x_{2i+2})\right) \leq V\left(\max\left\{u_1(x_{2i},x_{2i+1}),u_1(x_{2i+1},x_{2i+2})\right\}\right). \tag{2.2}$$

If $\max\{u_1(x_{2i},x_{2i+1}),u_1(x_{2i+1},x_{2i+2})\}) = u_1(x_{2i+1},x_{2i+2})$, then from (2.2), we have

$$V\left(u_1(x_{2i+1},x_{2i+2})\right) \leq V\left(u_1(x_{2i+1},x_{2i+2})\right) - \tau.$$

It is a contradiction. Therefore,

$$\max\left\{u_1(x_{2i},x_{2i+1}),u_1(x_{2i+1},x_{2i+2})\right\}) = u_1(x_{2i},x_{2i+1}), \text{ for all } i \in \{0,1,2,\dots\}.$$

Hence, from (2.2), we have

$$V\left(u_1(x_{2i+1},x_{2i+2})\right) \leq V\left(u_1(x_{2i},x_{2i+1})\right) - \tau. \tag{2.3}$$

Similarly, we have

$$V\left(u_1(x_{2i},x_{2i+1})\right) \leq V\left(u_1(x_{2i-1},x_{2i})\right) - \tau. \tag{2.4}$$

For all $i \in \{0,1,2,\dots.\}$. By (2.4) and (2.3), we have

$$V\left(u_1(x_{2i+1},x_{2i+2})\right) \leq V\left(u_1(x_{2i-1},x_{2i})\right) - 2\tau.$$

Repeating these steps, we get

$$V\left(u_1(x_{2i+1},x_{2i+2})\right) \leq V\left(u_1(x_0,x_1)\right) - (2i+1)\tau. \tag{2.5}$$

Similarly, we have

$$V\left(u_1(x_{2i},x_{2i+1})\right) \leq V\left(u_1(x_0,x_1)\right) - 2i\tau. \tag{2.6}$$

Inequalities (2.5) and (2.6) can jointly be written as

$$V\left(u_1(x_n,x_{n+1})\right) \leq V\left(u_1(x_0,x_1)\right) - n\tau. \tag{2.7}$$

Taking the limit as $n \to \infty$ in (2.7), we have

$$\lim_{n\to\infty} V\left(u_1(x_n,x_{n+1})\right) = -\infty.$$

Since $V \in R$, one gets

$$\lim_{n\to\infty} u_1(x_n,x_{n+1}) = 0. \tag{2.8}$$

Applying the property $(F1)$, we have some $k \in (0,1)$, for which

$$\lim_{n\to\infty}\big(u_1(x_n,x_{n+1})\big)^k\big(V\big(u_1(x_n,x_{n+1})\big) = 0. \tag{2.9}$$

By (2.7), for all $n \in \mathbb{N}$, we obtain

$$\big(u_1(x_n,x_{n+1})\big)^k\big(\big(V\big(u_1(x_n,x_{n+1})\big) - V\big(u_1(x_0,x_1)\big)\big) \leq -\big(u_1(x_n,x_{n+1})\big)^k_{n\tau} \leq 0. \tag{2.10}$$

Considering (2.8), (2.9) and letting $n \to \infty$ in (2.10), we have

$$\lim_{n\to\infty}\big(n\big(u_1(x_n,x_{n+1})\big)^k\big) = 0. \tag{2.11}$$

Since (2.11) holds, there exists $n_1 \in \mathbb{N}$ such that $n(u_1(x_n,x_{n+1}))^k \leq 1$ for all $n \geq n_1$, or

$$u_1(x_n,x_{n+1}) \leq \frac{1}{n^{\frac{1}{k}}} \quad \text{for all } n \geq n_1. \tag{2.12}$$

Take $p > 0$ and $m = n + p > n > n_1$, then

$$\begin{aligned} u_p(x_n,x_m) &\leq u_1(x_n,x_{n+1}) + u_1(x_{n+1},x_{n+2}) + \cdots + u_1(x_{m-1},x_m) \\ &\leq \frac{1}{n^{\frac{1}{k}}} + \frac{1}{(n+1)^{\frac{1}{k}}} + \cdots + \frac{1}{(m-1)^{\frac{1}{k}}}. \end{aligned}$$

As $k \in (0,1)$, then $\frac{1}{k} > 1$ and, by the ratio test,

$$\lim_{m,n\to\infty} u_p(x_n,x_m) = 0. \tag{2.13}$$

Since u satisfies the Δ_M- condition, we have

$$\lim_{m,n\to\infty} u_1(x_n,x_m) = 0. \tag{2.14}$$

Hence, the sequence $\{TS(x_n)\}$ is Cauchy in the complete regular modular-like type metric space $(\mathcal{L},u)$, hence there is $k \in \mathcal{L}$ so that $\{TS(x_n)\} \to k$ as $n \to \infty$. □

Theorem 2.3 *Let $(\mathcal{L},u)$ be a complete m.l.m. space. Assume that $S,T : \mathcal{L} \to W(\mathcal{L})$ are two fuzzy α_*-dominated mappings on $\{TS(x_n)\}$. If (S,T) is a rational fuzzy dominated V-contraction and k is the limit of the sequence $\{TS(x_n)\}$. If $\alpha(x_n,k) \geq 1 \forall n \in \{0,1,2,\dots\}$, then k belongs to both $[Tk]_{\beta(k)}$ and $[Sk]_{\gamma(k)}$.*

Proof As (S,T) is a rational fuzzy dominated V-contraction, then, by Theorem 2.2, there exists $k \in \mathcal{L}$ such that $\{TS(x_n)\} \to k$ as $n \to \infty$ and so

$$\lim_{n\to\infty} u_1(x_n,k) = u_1(k,k) = 0. \tag{2.15}$$

Now, by Lemma 1.10, we have

$$\tau + V\big(u_1\big(x_{2n+1},[Tk]_{\beta(k)}\big) \leq \tau + V\big(H_{u_1}\big([Sx_{2n}]\gamma_{x_{2n}},[Tk]_{\beta(k)}\big)\big). \tag{2.16}$$

By supposition, $\alpha(x_n, k) \geq 1$, so assume that $u_1(k, [Tk]_{\beta(k)}) > 0$, then there must be a positive natural number p so that $u_1(x_{2n+1}, [Tk]_{\beta(k)}) > 0$, for every $n \geq p$. Now, $H_{u_1}([Sx_{2n}]\gamma_{x_{2n}}, [Tk]_{\beta(k)}) > 0$, so inequality (2.1) implies for every $n \geq p$,

$$\begin{aligned}
&\tau + V(u_1(x_{2n+1}, [Tk]_{\beta(k)}) \\
&\quad \leq V\left(\max\left\{\begin{array}{c} u_1(x_{2n}, k), u_1(x_{2n}, [Sx_{2n}]\gamma_{x_{2n}}, \frac{u_1(x_{2n}, x_{2n+1}) + u_1(x_{2n+1}, [Tk]_{\beta(k)})}{2}), \\ \frac{u_1^2((x_{2n}), [Sx_{2n}]\gamma_{p_{2n}}).u_1(k, [Tk]_{\beta(k)})}{1 + u_1^2(x_{2n}, k)} \end{array}\right\}\right).
\end{aligned}$$

Letting $n \to \infty$ and using (2.15), we get

$$\begin{aligned}
\tau + V(u_1(x_{2n+1}, [Tk]_{\beta(k)}) \leq V(u_1(k, [Tk]_{\beta(k)}) &\leq V\left(\frac{u_1(k, [Tk]_{\beta(k)})}{2}\right) \\
&\leq V(u_1(k, [Tk]_{\beta(k)})).
\end{aligned}$$

Since V is strictly increasing, (2.16) implies that

$$u_1(k, [Tk]_{\beta(k)}) < u_1(k, [Tk]_{\beta(k)}).$$

This is a contradiction. So, our supposition is not true. Hence $u_1(k, [Tk]_{\beta(k)}) = 0$ or $k \in [Tk]_{\beta(k)}$. Similarly, by proceeding Lemma 1.10 and inequality (2.1), we can prove that

$$u_1(k, [Sk]_{\gamma(k)}) = 0, \quad \text{so } k \in ([Sk]_{\gamma(k)}).$$

Hence, S and T have a common fuzzy fixed point k in $\mathcal{L}$. □

Definition 2.4 Let $\mathcal{L}$ be a nonempty set, $\preccurlyeq$ be a partial order on $B \subseteq \mathcal{L}$. We say that $a \preccurlyeq B$ whenever for all $b \in B$, we have $a \preccurlyeq b$. A mapping $S : \mathcal{L} \to W(\mathcal{L})$ is said to be fuzzy $\preccurlyeq$-dominated on B, if $a \preccurlyeq [Sa]_\gamma$ for each $a \in \mathcal{L} and \gamma \in (0, 1]$.

We have the following result for multi fuzzy $\preccurlyeq$-dominated mappings on $\{TS(x_n)\}$ in an ordered complete *m.l.m.* space.

Theorem 2.5 *Let $(\mathcal{L}, \preccurlyeq, u)$ be an ordered complete m.l.m. space. Suppose that u is regular and the Δ_M-condition holds. Take $x_0 \in \mathcal{L}$ and let $S, T : \mathcal{L} \to W(\mathcal{L})$ be fuzzy dominated mappings on $\{TS(x_n)\}$. Suppose there exist $\tau > 0, \gamma(x), \beta(g) \in (0, 1]$ and $V \in R$ such that the following holds*:

$$\begin{aligned}
&\tau + V(H_{u_1}([Sx]_{\gamma(x)}, [Tg]_{\beta(g)})) \\
&\quad \leq V\left(\max\left\{\begin{array}{c} u_1(x, g), u_1(x, [Sx]_{\gamma(x)}), \frac{u_2[x, Tg]_{\beta(g)}}{2}, \\ \frac{u_1^2(x, [Sx]_{\gamma(x)}).u_1(g, [Tg]_{\beta(g)})}{1 + u_1^2(x, g)} \end{array}\right\}\right) \qquad (2.17)
\end{aligned}$$

whenever $x, g \in \{TS(x_n)\}$, with either $x \preccurlyeq g$ or $g \preccurlyeq x$, and $H_{u_1}([Sx]_{\gamma(x)}, [Tg]_{\beta(g)}) > 0$. Then $\{TS(x_n)\} \to k \in \mathcal{L}$. Also, if (2.17) holds for k, $x_n \preccurlyeq k$ and $k \preccurlyeq x_n$ for all $n \in \{0, 1, 2, \ldots\}$, then k belongs to both $[Tk]_{\beta(k)}$ and $[Sk]_{\gamma(k)}$.

Proof Let $\alpha : \mathcal{L} \times \mathcal{L} \to [0,+\infty)$ be a mapping defined by $\alpha(x,g) = 1$ for all $x \in \mathcal{L}$ with $x \preccurlyeq g$, and $\alpha(x,g) = 0$ for all other elements $x,g \in \mathcal{L}$. As S and T are the fuzzy prevalent mappings on $\mathcal{L}$, so $x \preccurlyeq [Sx]_{\gamma(x)}$ and $x \preccurlyeq [Tx]_{\beta(x)}$ for all $x \in \mathcal{L}$. This implies that $x \preccurlyeq b$ for all $b \in [Sx]_{\gamma(x)}$ and $x \preccurlyeq e$ for all $x \in [Tx]_{\beta(x)}$. So, $\alpha(x,b) = 1$ for all $b \in [Sx]_{\gamma(x)}$ and $\alpha(x,e) = 1$ for all $x \in [Tx]_{\beta(x)}$. This implies that

$$\inf\bigl\{\alpha(x,g) : g \in [Sx]_{\gamma(x)}\bigr\} = 1 \quad \text{and} \quad \inf\{\alpha(x,g) : g \in [Tx]_{\beta(x)} = 1.$$

Hence, $\alpha_*(x,[Sx]_{\gamma(x)}) = 1, \alpha_*(x,[Tx]_{\beta(x)}) = 1$ for all $x \in \mathcal{L}$. So, $S,T : \mathcal{L} \to W(\mathcal{L})$ are α_*-dominated mappings on $\mathcal{L}$. Moreover, inequality (2.17) holds and it can be written as

$$\tau + V(H_{u_1}\bigl([Sx]_{\gamma(x)},[Tg]_{\beta(g)}\bigr) \le V\bigl(u_1(x,g)\bigr),$$

for all elements x,g in $\{TS(x_n)\}$, with either $\alpha(x,g) \ge 1$ or $\alpha(g,x) \ge 1$. Then, by Theorem 2.2, $\{TS(x_n)\}$ is a sequence in $\mathcal{L}$ and $\{TS(x_n)\} \to x^* \in \mathcal{L}$. Now, $x_n, x^* \in \mathcal{L}$ and either $x_n \preccurlyeq x^*$, or $x^* \preccurlyeq x_n$ implies that either $\alpha(x_n,x^*)$, or $\alpha(x^*,x_n) \ge 1$. So, all requirements of Theorem 2.3 are satisfied. Hence, x^* is the common fuzzy fixed point of both S and T in $\mathcal{L}$ and $u_l(x^*,x^*) = 0$. □

Example 2.6 Let $\mathcal{L} = Q^+ \cup \{0\}$ and $u_l(e,x) = \frac{1}{l}(e+x)$. Now,

$$u_2(e,x) = \frac{1}{2}(e+x) \quad \text{and} \quad u_1(e,x) = e+x,$$

$\forall e,x \in \mathcal{L}$. Define $S,T : \mathcal{L} \longrightarrow W(\mathcal{L})$ by

$$(Se)(t) = \begin{cases} \gamma & \text{if } \frac{g}{4} \le t < \frac{g}{2}, \\ \frac{\gamma}{2} & \text{if } \frac{g}{2} \le t \le \frac{3g}{4}, \\ \frac{\gamma}{4} & \text{if } \frac{3g}{4} < t \le g, \\ 0 & \text{if } g < t < \infty, \end{cases} \quad \text{and} \quad (Tx)(t) = \begin{cases} \beta & \text{if } \frac{g}{3} \le t < \frac{g}{2}, \\ \frac{\beta}{4} & \text{if } \frac{g}{2} \le t \le \frac{2g}{3}, \\ \frac{\beta}{6} & \text{if } \frac{2g}{3} < t \le g, \\ 0 & \text{if } g < t < \infty. \end{cases}$$

Now, we consider

$$[Se]_{\frac{\gamma}{2}} = \left[\frac{e}{4}, \frac{3e}{4}\right] \quad \text{and} \quad [Tx]_{\frac{\beta}{4}} = \left[\frac{x}{3}, \frac{2x}{3}\right].$$

Taking $e_0 = \frac{1}{2}$, then we have

$$u_1\bigl(e_0,[Se_0]_{\frac{\gamma}{2}}\bigr) = u_1\left(\frac{1}{2}, \left[\frac{1}{8}, \frac{3}{8}\right]\right) = u_1\left(\frac{1}{2}, \frac{1}{8}\right).$$

So, we obtain a sequence $\{TS(e_n)\} = \{\frac{1}{2}, \frac{1}{8}, \frac{1}{24}, \frac{1}{96}, \ldots\}$ in $\mathcal{L}$ generated by e_0.

$$\alpha(e,x) = \begin{cases} 1 & \text{if } e,x \in \mathcal{L}, \\ \frac{1}{2} & \text{otherwise.} \end{cases}$$

Now, for all $e,x \in \{TS(c_n)\}$ with either $\alpha(e,x) \geq 1$ or $\alpha(x,e) \geq 1$, we have

$$\begin{aligned}
&H_{u_1}\big([Se]_{\frac{\gamma}{2}}, [Tx]_{\frac{\beta}{4}}\big)\\
&= \max\Big\{\sup_{a\in[Se]_{\frac{\gamma}{2}}} u_1\big(a,[Tx]_{\frac{\beta}{4}}\big), \sup_{q\in[Tx]_{\frac{\beta}{4}}} u_1\big([Se]_{\frac{\gamma}{2}}, q\big)\Big\}\\
&= \max\left\{\sup_{a\in[\frac{e}{4},\frac{3e}{4}]} u_1\left(a,\left[\frac{x}{3},\frac{2x}{3}\right]\right), \sup_{q\in[\frac{x}{3},\frac{2x}{3}]} u_1\left(\left[\frac{e}{4},\frac{3e}{4}\right],q\right)\right\}\\
&= \max\left\{u_1\left(\frac{3e}{4},\left[\frac{x}{3},\frac{2x}{3}\right]\right), u_1\left(\left[\frac{e}{4},\frac{3e}{4}\right],\frac{2x}{3}\right)\right\}\\
&= \max\left\{u_1\left(\frac{3e}{4},\frac{x}{3}\right), u_1\left(\frac{e}{4},\frac{2x}{3}\right)\right\} = \max\left\{\frac{3e}{4}+\frac{x}{3}, \frac{e}{4}+\frac{2x}{3}\right\}.\\
&= \max\left\{u_1(x,e), u_1\big(x,[Sx]_{\gamma(x)}\big), \frac{u_2(x,[Te]_{\beta(e)}}{2}, \frac{u_1^2(x,[Sx)_{\gamma(x)}).u_1(e,[Te]_{\beta(e)})}{1+u_1^2(x,e)}\right\}\\
&= \max\left\{(x,e), \left(e+\frac{e}{4}\right), \frac{1}{2}\left(e+\frac{x}{3}\right), \frac{(e+\frac{e}{4})^2.(x+\frac{x}{3})}{1+(e+x)^2}\right\} = e+x.
\end{aligned}$$

i. Case (1): If $\max\{(\frac{3e}{4}+\frac{x}{3}),(\frac{e}{4}+\frac{2x}{3})\} = (\frac{3e}{4}+\frac{x}{3})$ and $\tau = \ln(1.2)$, then we have

$$\frac{9e}{2} + 2x \leq 5e + 5x.$$

Then

$$\frac{6}{5}\left(\frac{3e}{4}+\frac{x}{3}\right) \leq e + x.$$

Therefore,

$$\ln(1.2) + \ln\left(\frac{3e}{4}+\frac{x}{3}\right) \leq \ln(e+x).$$

This implies that

$$\begin{aligned}
&\tau + V\big(H_{u_1}\big([Se]_{\frac{\gamma}{2}}, [Tx]_{\frac{\beta}{4}}\big)\big)\\
&\quad\leq V\left(\max\left\{\begin{array}{c} u_1(x,e), u_1(x,[Sx]_{\gamma(x)}),\\ \frac{u_2(x,[Te]_{\beta(e)})}{2}. \frac{u_1^2(x,[Sx]_{\gamma(x)}).u_1(e,[Te]_{\beta(e)})}{1+u_1^2(x,e)}\end{array}\right\}\right)
\end{aligned}$$

Figure 1 illustrates Case (i) of Example 2.6, where the graph in blue represents the right side of the contractive inequality of Theorem 2.3 and that in red shows the left side of the inequality of Theorem 2.3.

ii. Case (ii): If $\max\{(\frac{3e}{4}+\frac{x}{3}),(\frac{e}{4},\frac{2x}{3})\} = (\frac{e}{4}+\frac{2x}{3})$ and $\tau = \ln(1.2)$, then we have

$$\frac{3e}{2} + 4x \leq 5e + 5x.$$

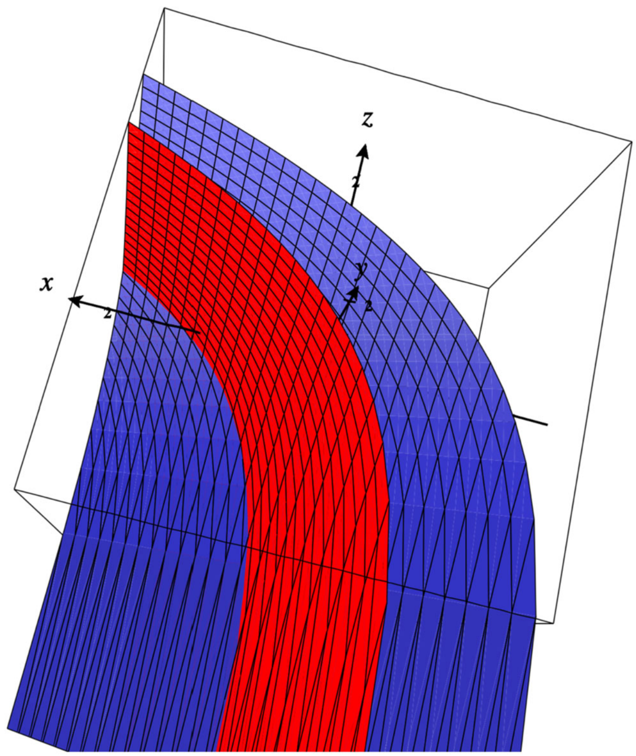

Figure 1 Comparison of the functions $V(H_{u_1}([Se]_{\frac{\gamma}{2}},[Tx]_{\frac{\beta}{4}}))$ and $E(N(e,x))$. $= V(H_{u_1}([Se]_{\frac{\gamma}{2}},[Tx]_{\frac{\beta}{4}}))$; $= E(N(e,x))$

That is,

$$\frac{6}{5}\left(\frac{e}{4}+\frac{2x}{3}\right)\leq e+x.$$

Then

$$\ln(1.2)+\ln\left(\frac{e}{4}+\frac{2x}{3}\right)\leq \ln(e+x).$$

This implies that

$$\tau + V\bigl(H_{u_1}\bigl([Se]_{\frac{\gamma}{2}},[Tx]_{\frac{\beta}{4}}\bigr)\bigr)\leq V\left(\max\left\{\begin{array}{c} u_1(x,e),u_1(x,[Sx]_{\gamma(x)}), \\ \frac{u_2(x,[Te]_{\beta(e)})}{2},\frac{u_1^2(\beta,[Sx]_{\gamma(x)}).u_1(e,[Te]_{\beta(e)})}{1+u_1^2(x,e)} \end{array}\right\}\right)$$

Figure 2 illustrates Case (ii) of Example 2.6, where the graph in blue represents the right side of the contractive inequality of Theorem 2.3 and that in red shows the left side of the inequality of Theorem 2.3 (See Figure 3).

Hence, all requirements of Theorem 2.3 are satisfied.

Corollary 2.7 *Let $(\mathcal{L},u)$ be a complete m.l.m. space. Assume that u is regular and the Δ_M-condition holds. Take $x_0\in\mathcal{L},\alpha:\mathcal{L}\times\mathcal{L}\longrightarrow[0,\infty)$ and let $S:\mathcal{L}\longrightarrow W(\mathcal{L})$ be a fuzzy*

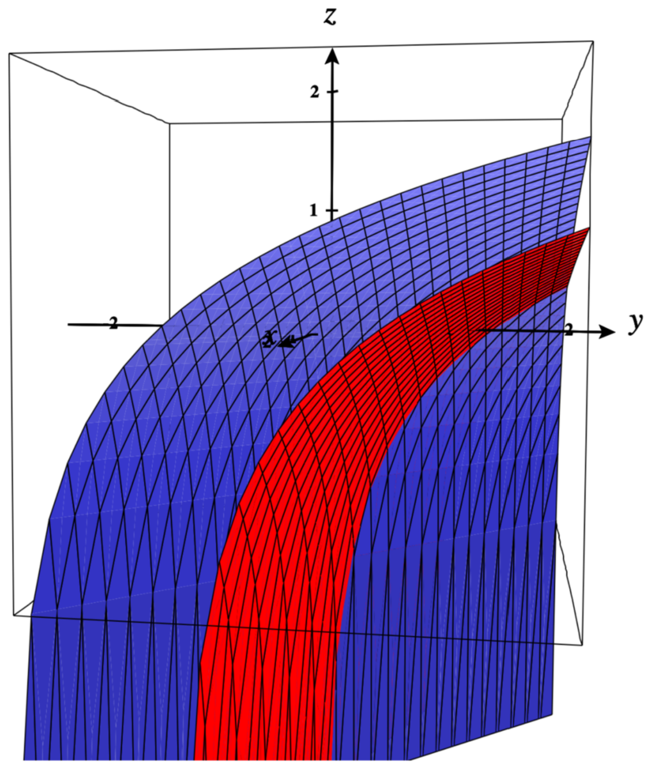

Figure 2 Comparison of the functions $E(H_{u_1}([Se]_{\frac{\gamma}{2}},[Tx]_{\frac{\beta}{4}}))$ and $E(N(e,x))$. = $E(H_{u_1}([Se]_{\frac{\gamma}{2}},[Tx]_{\frac{\beta}{4}}))$; = $E(N(e,p))$

α_*-dominated mapping on $\{SS(x_n)\}$. Suppose there exist $\tau > 0, \gamma(x), \beta(g) \in (0,1]$, and $V \in R$ such that

$$\begin{aligned} &\tau + V\bigl(H_{u_1}\bigl([Sx]_{\gamma(x)},[Sg]_{\beta(g)}\bigr)\bigr) \\ &\quad \leq V\left(\max\left\{\begin{array}{c} u_1(x,g), u_1(x,[Sx]_{\gamma(x)}, \frac{u_2(x,[Sg]_{\beta(g)})}{2}, \\ \frac{u_1^2(x,[Sx]_{\gamma(x)}).u_1(g,[Sg]_{\beta(g)})}{1+u_1^2(x,g)} \end{array}\right\}\right), \end{aligned} \tag{2.18}$$

whenever $x, g \in \{SS(x_n)\}, \alpha(x,g) \geq 1$ and $H_{u_1}([Sx]_{\gamma(x)},[Sg]_{\beta(g)}) > 0$. Then $\alpha(x_n,x_{n+1}) \geq 1$ for all $n \in \{0,1,2,\dots\}$ and $\{SS(x_n)\} \to k \in \mathcal{L}$. Also, if either $\alpha(x_n,k) \geq 1$ or $\alpha(k,x_n) \geq 1$ for all $n \in \{0,1,2\dots\}$, then $k \in [Sk]_{\gamma(k)}$.

If we take multivalued α_*-dominated mappings from a ground set $\mathcal{L}$ to the proximinal subsets of $\mathcal{L}$ instead of fuzzy α_*-dominated mappings from $\mathcal{L}$ to the approximate quantities $W(\mathcal{L})$ in Theorem 2.3, we obtain the following result.

Corollary 2.8 *Let $(\mathcal{L},u)$ be a complete modular-like metric space. Assume that u is regular and verifies the Δ_{M-} condition. Let $x_0 \in \mathcal{L}, \alpha : \mathcal{L} \times \mathcal{L} \to [0,\infty)$ and $S,T : \mathcal{L} \to W(\mathcal{L})$ be two*

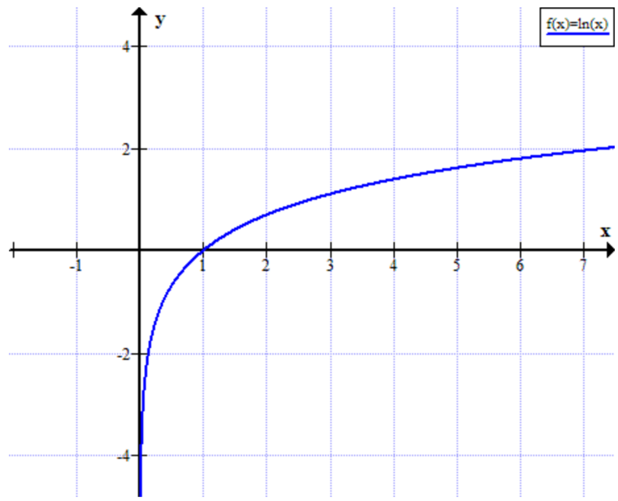

Figure 3 Graph for $\ln x$. ln(x)

α_*-dominated mappings on $\{TS(x_n)\}$. Suppose there exist $\tau > 0$ and $V \in R$ such that

$$\tau + V\big(H_{u_1}(Sx, Tg)\big) \leq V\left(\max\left\{\begin{matrix} u_1(x,g), u_1(x,Sx), \frac{u_2(x,Tg)}{2}, \\ \frac{u_1^2(x,Sx).u_1(g,Tg)}{1+u_1^2(x,g)} \end{matrix}\right\}\right) \tag{2.19}$$

whenever $x, g \in \{TS(x_n)\}, \alpha(x,g) \geq 1$, and $H_{u_1}(Sx, xg) > 0$. Then, $\alpha(x_n, x_{n+1}) \geq 1$ for all $n \in \{0,1,2,\dots\}$ and $\{TS(x_n)\} \to k \in \mathcal{L}$. Also, if either $\alpha(x_n,k) \geq 1$ or $\alpha(k,x_n) \geq 1$ for all $n \in \{0,1,2\dots\}$, then k belongs to both Tk and Sk.

If we take $S = T$ in Corollary 2.8, we obtain the following result.

Corollary 2.9 *Let $(\mathcal{L}, u)$ be a complete modular-like metric space metric space. Assume that u is regular and satisfies the Δ_M-condition. Let $x_0 \in \mathcal{L}, \alpha : \mathcal{L} \times \mathcal{L} \to [0,\infty)$ and $S : \mathcal{L} \to W(\mathcal{L})$ be a multivalued α_*-dominated mapping on $\{SS(x_n)\}$. Suppose there exist $\tau > 0$ and $V \in R$ such that*

$$\tau + V\big(H_{u_1}(Sx, Sg)\big) \leq V\left(\max\left\{\begin{matrix} u_1(x,g), u_1(x,Sx), \frac{u_2(x,Sg)}{2}, \\ \frac{u_1^2(x,Sx).u_1(g,Sg)}{1+u_1^2(x,g)} \end{matrix}\right\}\right) \tag{2.20}$$

whenever $x, g \in \{SS(x_n)\}, \alpha(x,g) \geq 1$, and $x_{u_1}(Sx, Sg) > 0$. Then $\alpha(x_n, x_{n+1}) \geq 1$ for all $n \in \{0,1,2,\dots\}$ and $\{SS(x_n)\} \to k \in \mathcal{L}$. Also, if either $\alpha(x_n,k) \geq 1$ or $\alpha(k,x_n) \geq 1$ for all $n \in \{0,1,2,\dots\}$, then k belongs to Sk.

3 Applications on graphic contractions

Jachymski [30] proved a relation between graph and fixed point theory by the orientation of graphic contractions. Let A be a nonempty set. Let $V(Y)$ and $L(Y)$ denote the set of vertices and the set of edges containing all loops, respectively, for a graph Y.

Definition 3.1 Let A be a nonempty set and $Y = (V(Y), L(Y))$ be a graph with $A = V(Y)$. A fuzzy map F from A to $W(A)$ is called fuzzy-graph dominated on A if $(a,b) \in L(Y)$, for $a \in A$, $b \in [Fa]_\beta$ and $0 < \beta \le 1$.

Theorem 3.2 *Let $(\mathcal{L}, u)$ be a complete m.l.m. space equipped with a graph $Y, x_0 \in \mathcal{L}$ so that*

(i) $S, T : \mathcal{L} \to W(\mathcal{L})$ are fuzzy-graph dominated functions on $\{TS(p_n)\}$;

(ii) $\tau + V(H_{u_1}([St]_{\gamma(t)}, [Ty]_{\beta(y)}))$, and $\tau > 0, \gamma(t), \beta(y)$ in $[0,1]$

$$\le V\left(\max\left\{u_1(t,y), u_1\left(t, [St]_{\gamma(t)}\right), \frac{u_2(t, [Ty]_{\beta(y)})}{2}, \frac{u_1^2(t, [St]_{\gamma(t)}).u_1(y, [Ty]_{\beta(y)}}{1 + u_1^2(t,y)}\right\}\right) \tag{3.1}$$

whenever $t, y \in \{TS\}(\boldsymbol{x_n})\}, (t,y) \in L(Y)$, and $H_{u1}([St]_{\gamma(t)}, [Ty]_{\beta(y)}) > 0$. Suppose that $\mathcal{L}$ is regular and the $\triangle_M$-condition holds. Then $(x_n, x_{n+1}) \in L(Y)$ and $\{TS(x_n)\} \to k^*$. Also, if $(x_n, k^*) \in L(Y)$ or $(k^*, x_n) \in L(Y)$ for each $n \in \{0, 1, 2, \dots\}$, then k^* belongs to both $[Tk^*]_{\beta(k^*)}$ and $k \in [Sk^*]_{\gamma(k^*)}$.

Proof Define $\alpha : \mathcal{L} \times \mathcal{L} \to [0, \infty)$ by $\alpha(t,y) = 1$, if $t \in \mathcal{L}$ and $(t,y) \in L(Y)$. Otherwise, take $\alpha(t,y) = 0$. The graph dominated notion on $\mathcal{L}$ gives that $(t,y) \in L(Y)$ for all $y \in [St]_{\gamma(t)}$ and $(t,y) \in L(Y)$ for each $y \in [Ty]_{\beta(y)}$. So, $\alpha(t,y) = 1$ for all $y \in [St]_{\gamma(t)}$ and $\alpha(t,y) = 1$ for every $y \in [Ty]_{\beta(y)}$. This means that

$$\inf\{\alpha(t,y) : y \in [St]_{\gamma(t)} = 1 \quad \text{and} \quad \inf\left\{\alpha(t,y) : y \in [Ty]_{\beta(y)}\right\} = 1.$$

Hence, $\alpha_*(t, [St]_{\gamma(t)}) = 1, \alpha_*(t, [Ty]_{\beta(y)}) = 1$, for every $t \in \mathcal{L}$. So, the mappings are α_*-dominated on $\mathcal{L}$. Furthermore, inequality (3.1) can be expressed as

$$\tau + V\left(H_{u_1}\left([St]_{\gamma(t)}, [Ty]_{\beta(y)}\right)\right) \le V\left(\max\left\{\begin{array}{c} u_1(t,y), u_1(t, [St]_{\gamma(t)}), \\ \frac{u_2(t,[Ty]_{\beta(y)})}{2}, \frac{u_1^2(t,[St]_{\gamma(t)}).u_1(y,[Ty]_{\beta(y)}}{1+u_1^2(t,y)} \end{array}\right\}\right)$$

whenever $t, y \in \{TS(x_n)\}, \alpha(t,y) \ge 1$ and $H_{u_1}([St]_{\gamma(t)}, [Ty]_{\beta(y)}) > 0$. Also, (ii) holds. Using Theorem 2.2, we have $\{TS(x_n)\}$ is a sequence in $\mathcal{L}$ and $\{TS(x_n)\} \to k^* \in \mathcal{L}$. Now, $x_n, k^* \in \mathcal{L}$ and either $(x_n, k^*) \in L(Y)$, or $(k^*, x_n) \in L(Y)$ implies that either $\alpha(x_n, k^*) \ge 1$ or $\alpha(k^*, x_n) \ge 1$. So,all conditions of Theorem 2.2 are checked. Hence, k^* belongs to both $[Tk^*]_{\beta(k^*)}$ and $k \in [Sk^*]_{\gamma(k^*).}$ □

4 Applications to electric circuit equations

In this section, we discuss the solution of the electric circuit equation (see [7]) which is a second-order differential equation. The electric circuit (as in Fig. 4) contains an electromotive force E, a resistor R, an inductor L, a capacitor C, and a voltage V in series. If the current I is the rate of change of q with respect to time t, we have $I = \frac{dq}{dt}$ and

$$V = IR,$$

$$V = qC,$$

$$V = L\frac{dI}{dt}.$$

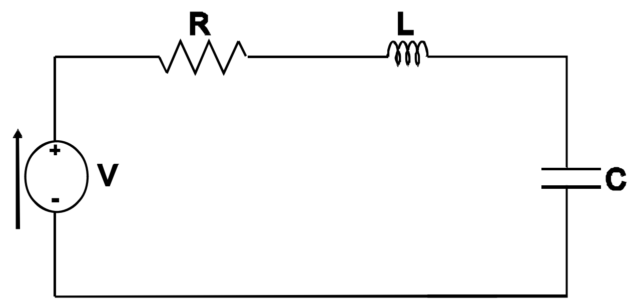

Figure 4 Electric circuit

By Kirchhoff's law, the sum of these voltage drops is equal to supplied voltage, i.e.,

$$IR + \frac{q}{c} + L\frac{dI}{dt} = V(t),$$

or

$$IR + \frac{q}{c} + L\frac{dI}{dt} = V(t), \qquad q(0) = 0, \qquad q'(0) = 0. \tag{ECE}$$

The Green function associated to (ECE) is given by

$$G(t,s) = \begin{cases} -se^{\tau(s-t)} & \text{if } 0 \le s \le t \le 1, \\ -te^{\tau(s-t)} & \text{if } 0 \le t \le s \le 1, \end{cases}$$

where the constant $\tau > 0$ is calculated in terms of R and L. Let $\mathcal{L} = C[0,1]$ be the set of all continuous functions defined on $[0,1]$. The modular-like metric u on $\mathcal{L}$ is defined as

$$u(t,g) = \frac{1}{2} \sup_{k \in [0,1]} \{ |t(k) + g(k)| e^{-\tau k} \} = \frac{1}{2} \|t + g\|_{\tau}.$$

Moreover, we define the graph with the partial order relation: for $u, g \in C[0,1]$,

$$u \le g \leftrightarrow u(t) \le g(t)$$

for all $t \in [0,1]$. Let $Y(G) = \{(u,g) \in \mathcal{L} \times \mathcal{L} : u \le g\}$. Note that $(u, \mathcal{L})$ is a complete modular-like metric space, including a direct graph G; $\Delta = (\mathcal{L} \times \mathcal{L}) \in Y(G)$ and $(u, \mathcal{L}, G)$ has a property (E^*).

Theorem 4.1 *Let $S, T : C[0,1] \to C[0,1]$ be self-mappings of the modular-like metric space $(C[0,1), u)$. Assume that*

(i) *There exist continuous and nondecreasing functions $H, Q : C[0,1] \times \mathbb{R} \to \mathbb{R}$ such that for all $b, c \in C[0,1]$, with $b \le c$, there exists $\tau > 0$ so that*

$$H(t, b(s)) + Q(t, c(s)) \le \frac{\tau E(b,c)}{\tau E(b,c) + 1},$$

where

$$E(b,c)=\max\left(\frac{1}{2}\left\{\begin{array}{c}\|b+c\|_\tau,\|b+Sb\|_\tau,\frac{\|b+Sb\|_\tau+\|c+Tc\|_\tau}{2},\\ \frac{\|b+Sb\|_\tau^2.\|c+Tc\|_\tau}{1+\|b+c\|_\tau^2}\end{array}\right\}\right)$$

for all $t,s\in[0,1]$ *and* $b,c\in C([0,1],\mathbb{R}^+)$.

(ii) *There are* b_0, $c_0\in C([0,1])$

$$b_0(t)\le\int_0^t G(t,s)H\big(t,b_0(s)\big)\,ds\quad\text{for all } t\in[0,1]$$

and

$$c_0\le\int_0^t G(t,s)Q\big(t,c_0(s)\big)\,ds\quad\text{for all } t\in[0,1].$$

Then the differential equation arising in the electric circuit (ECE) *has a solution.*

Proof The problem (ECE) is equivalent to integral forms given as

$$b(t)=\int_0^1 G(t,s)H\big(t,b(s)\big)\,ds \tag{4.1}$$

and

$$c(t)=\int_0^1 G(t,s)Q\big(t,c(s)\big)\,ds, \tag{4.2}$$

where $t\in[0,1]$. Consider $S,T:\mathcal{L}\to\mathcal{L}$ defined by

$$(Sb)(t)=\int_0^t G(t,s)H\big(t,b(s)\big)\,ds \tag{4.3}$$

and

$$(Tc)(t)=\int_0^t G(t,s)Q\big(t,c(s)\big)\,ds, \tag{4.4}$$

where $t\in C[0,1]$. Then b^* is the solution of (4.1) and (4.2) if and only if b^* is a common fixed point of S and T. From condition (ii), it is very easy to show that for every $u,g\in\mathcal{L}$, we have $u\le Su$ and $g\le Tg$, i.e.,

$$\big(b,S(b)\big)\in Y(G)\neq\emptyset\quad\text{and}\quad\big(g,T(g)\big)\in Y(G)\neq\emptyset.$$

Let $b,g\in\mathcal{L}$, then from condition (i), we have

$$\begin{aligned}|Sb+Tc|&\le\int_0^t G(t,s)\big|H\big(t,b(s)\big)+Q\big(t,c(s)\big)\big|\,ds\le\int_0^t\frac{\tau E(b,c)}{\tau E(b,c)+1}e^{-\tau s}G(t,s)\,ds\\&\le\frac{\tau E(b,c)}{\tau E(b,c)+1}\int_0^t e^{-\tau s}G(t,s)\,ds\le\frac{\tau E(b,c)}{\tau E(b,c)+1}e^{\tau t}\big[1-2tr+t\tau e^{-\tau t}-e^{-\tau t}\big].\end{aligned}$$

This implies that

$$|Sb + Tc|e^{\tau t} \leq \frac{\tau E(b,c)}{\tau E(b,c) + 1}\left[1 - 2tr + t\tau e^{-\tau t} - e^{-\tau t}\right].$$

Since $1 - 2tr + t\tau e^{-\tau t} - e^{-\tau t} \leq 1$, we get that

$$\begin{aligned}
&\|Sb + Tc\|_\tau \leq \frac{\tau E(b,c)}{\tau E(b,c) + 1}, \\
&\frac{\tau E(b,c) + 1}{\tau E(b,c)} \leq \frac{1}{\|Sb + Tc\|_\tau}, \\
&\tau + \frac{1}{E(b,c)} \leq \frac{1}{\|Sb + Tc\|_\tau},
\end{aligned}$$

which further implies

$$\tau - \frac{1}{\|Sb + Tc\|_\tau} \leq \frac{-1}{E(b,c)}.$$

So, all the requirements of Theorem 2.2 are satisfied for $R(f) = \frac{-1}{f}, f > 0$, and $u(b,c) = \frac{1}{2}\|b + c\|_\tau$. Hence, the mappings S and T have a common fixed point. Consequently, the differential equation arising in the electric circuit (ECE) has a solution. □

5 Applications to fractional differential equations

Lacroix (1819) established and proved many important properties of fractional differentials. Later, many authors proved some new fixed-point results involving their applications related to fractional differential and integral equations, see [4, 7, 34]. Recently, a large number of new models relevant to Caputo–Fabrizio derivative (CFD) were introduced and investigated, see [15, 40, 65, 69–72]. In this section, we investigate one of these models in modular-like metric spaces.

Let $C[0,1]$ be the space of continuous functions. Consider

$$d(w,g) = \frac{|w + g|}{2} \quad \text{for all } w,g \in C[0,1].$$

The space $(C[0,1], d)$ is a complete modular-like metric space and $V(t) = \ln t$.

Let $\mathcal{K}_1, \mathcal{K}_2 : [0,1] \times R \to R$ be continuous mappings. We will investigate the CFD equations:

$$D^\beta q(\nu) = \mathcal{K}_1\bigl(\nu, q(\nu)\bigr) \tag{5.1}$$

with boundary conditions $q(0) = 0, Iq(1) = q'(0)$, and

$$D^\beta g(u) = \mathcal{K}_2\bigl(u, g(u)\bigr) \tag{5.2}$$

with boundary conditions $g(0) = 0, Ig(1) = g'(0)$.

Here, D^β is the CFD of order β defined by

$$D^\beta K_1(\nu) = \frac{1}{\Gamma(n-\beta)} \int_0^\nu (\nu - n)^{n-\beta-1} \mathcal{K}_1^n(\nu)\, d\nu$$

where $n-1<\beta<n$ and $n=[\beta]+1$, and $I^{\beta}\mathcal{K}_1$ is given by

$$I^{\beta}\mathcal{K}_1(\nu)=\frac{1}{\Gamma(\beta)}\int_0^{\nu}(\nu-n)^{\beta-1}\mathcal{K}_1(\nu)\,d\nu \quad \text{with } \beta>0.$$

Then Eq. (5.1) can be modified to

$$q(\nu)=\frac{1}{\Gamma(\beta)}\int_0^{\nu}(\nu-n)^{\beta-1}\mathcal{K}_1\bigl(\omega,q(\omega)\bigr)\,d\omega+\frac{2\nu}{\Gamma(\beta)}\int_0^1\int_0^{\omega}(n-u)^{\beta-1}\mathcal{K}_1\bigl(u,q(u)\bigr)\,du\,d\omega.$$

Similarly, Eq. (5.2) can be modified to

$$g(l)=\frac{1}{\Gamma(\beta)}\int_0^{l}(l-n)^{\beta-1}\mathcal{K}_2\bigl(\omega,g(\omega)\bigr)\,d\omega+\frac{2l}{\Gamma(\beta)}\int_0^1\int_0^{\omega}(n-p)^{\beta-1}\mathcal{K}_2\bigl(p,q(p)\bigr)\,dp\,d\omega.$$

Theorem 5.1 *Suppose that*:

(*I*) *there exists* $\tau>0$ *such that for all* $e,s\in C[0,1]$, *we have*

$$\bigl|\mathcal{K}_1\bigl(u,e(u)\bigr)\,du+\mathcal{K}_2\bigl(u,s(u)\bigr)\,du\bigr|\le\frac{e^{-\tau}\Gamma(\beta+1)}{4V}\bigl|e(u)+s(u)\bigr|;$$

(*II*) *there exist* $h,g\in C[0,1]$ *such that for every* $\nu,z\in C[0,1]$,

$$h(\nu)=\frac{1}{\Gamma(\beta)}\int_0^{\nu}(\nu-n)^{\beta-1}\mathcal{K}_1\bigl(\omega,q(\omega)\bigr)\,d\omega+\frac{2\nu}{\Gamma(\beta)}\int_0^1\int_0^{\omega}(n-u)^{\beta-1}\mathcal{K}_1\bigl(u,q(u)\bigr)\,du\,d\omega$$

and

$$g(z)=\frac{1}{\Gamma(\beta)}\int_0^{\nu}(\nu-n)^{\beta-1}\mathcal{K}_2\bigl(\omega,q(\omega)\bigr)\,d\omega+\frac{2\nu}{\Gamma(\beta)}\int_0^1\int_0^{\omega}(n-u)^{\beta-1}\mathcal{K}_2\bigl(u,q(u)\bigr)\,du\,d\omega.$$

Then Eqs. (5.1) *and* (5.2) *have a solution in* $C[0,1]$.

Proof Define the mappings $S,T:C[0,1]\to C[0,1]$ by

$$\begin{aligned}S\bigl(q(\nu)\bigr)&=\frac{1}{\Gamma(\beta)}\int_0^{\nu}(\nu-n)^{\beta-1}\mathcal{K}_1\bigl(\omega,q(\omega)\bigr)\,d\omega\\&\quad+\frac{2\nu}{\Gamma(\beta)}\int_0^1\int_0^{\omega}(n-u)^{\beta-1}\mathcal{K}_1\bigl(u,q(u)\bigr)\,du\,d\omega\end{aligned}$$

and

$$\begin{aligned}T\bigl(g(z)\bigr)&=\frac{1}{\Gamma(\beta)}\int_0^{\nu}(\nu-n)^{\beta-1}\mathcal{K}_2\bigl(\omega,q(\omega)\bigr)\,d\omega\\&\quad+\frac{2\nu}{\Gamma(\beta)}\int_0^1\int_0^{\omega}(n-u)^{\beta-1}\mathcal{K}_2\bigl(u,q(u)\bigr)\,du\,d\omega.\end{aligned}$$

By (II), there exist $h,g\in C[0,1]$ such that $h_n=S^n(h)$ and $g_n=T^n(g)$. The continuity of $\mathcal{K}_1$ and $\mathcal{K}_2$ leads to the continuity of the mappings S and T on C[0,1]. It is easy to verify the assumptions of Theorem 2.2 hold. For this, we have that

$$\bigl|S\bigl(q(\nu)\bigr)+T\bigl(g(z)\bigr)\bigr|$$

$$= \left| \begin{array}{c} \frac{1}{\Gamma(\beta)} \int_0^{\nu} (\nu-n)^{\beta-1} \mathcal{K}_1(\omega, q(\omega))\, d\omega + \frac{1}{\Gamma(\beta)} \int_0^{\nu} (\nu-n)^{\beta-1} \mathcal{K}_2(\omega, q(\omega))\, d\omega \\ + \frac{2\nu}{\Gamma(\beta)} \int_0^1 \int_0^{\omega} (n-u)^{\beta-1} \mathcal{K}_1(u, q(u))\, du\, d\omega \\ + \frac{2\nu}{\Gamma(\beta)} \int_0^1 \int_0^{\omega} (n-u)^{\beta-1} \mathcal{K}_2(u, q(u))\, du\, d\omega \end{array} \right| \quad \text{implies}$$

$$\begin{aligned}
&\bigl|S\bigl(q(\nu)\bigr) + T\bigl(g(z)\bigr)\bigr| \\
&\leq \left| \int_0^{\nu} \left(\frac{1}{\Gamma(\beta)} (\nu-n)^{\beta-1} \mathcal{K}_1\bigl(\omega, q(\omega)\bigr) + \frac{1}{\Gamma(\beta)} (\nu-n)^{\beta-1} \mathcal{K}_2\bigl(\omega, q(\omega)\bigr) \right) d\omega \right| \\
&\quad + \left| \int_0^1 \int_0^{\omega} \left(\frac{2\nu}{\Gamma(\beta)} (n-u)^{\beta-1} \mathcal{K}_1\bigl(u, q(u)\bigr) + \frac{2\nu}{\Gamma(\beta)} (n-u)^{\beta-1} \mathcal{K}_2\bigl(u, q(u)\bigr) \right) du\, d\omega \right| \\
&\leq \frac{1}{\Gamma(\beta)} . \frac{e^{-\tau}\Gamma(\beta+1)}{4V} . \int_0^{\nu} (\nu-n)^{\beta-1} \bigl(h(z) + g(z)\bigr)\, dz \\
&\quad + \frac{2}{\Gamma(\beta)} . \frac{e^{-\tau}\Gamma(\beta+1)}{4V} . \int_0^1 \int_0^{\omega} (n-u)^{\beta-1} \bigl(h(u) + g(u)\bigr)\, du\, d\omega \\
&\leq \frac{1}{\Gamma(\beta)} . \frac{e^{-\tau}\Gamma(\beta+1)}{4V} .d(h,g). \int_0^{\nu} (\nu-n)^{\beta-1}\, dz \\
&\quad + \frac{2}{\Gamma(\beta)} . \frac{e^{-\tau}\Gamma(\beta+1).\Gamma(\beta+1)}{\Gamma(s).4V.\Gamma(\beta+1)} .d(h,g). \int_0^1 \int_0^{\omega} (n-u)^{\beta-1}\, du\, d\omega \\
&\leq \left(\frac{e^{-\tau}\Gamma(\beta).\Gamma(\beta+1)}{\Gamma(\beta).4V.\Gamma(\beta+1)} \right) .d(h,g) + 2e^{-\tau} B(\beta+1, 1) \frac{\Gamma(\beta).\Gamma(\beta+1)}{\Gamma(\beta).4V.\Gamma(\beta+1)} .d(h,g) \\
&\leq \frac{e^{-\tau}}{4V} d(h,g) + \frac{e^{-\tau}}{2V} d(h,g) < \frac{e^{-\tau}}{V} d(h,g),
\end{aligned}$$

where B is the beta mapping. The last inequality can be written as

$$V\bigl|S\bigl(h(\nu)\bigr) + T\bigl(g(z)\bigr)\bigr| \leq e^{-\tau} d(h,g) \leq e^{-\tau} E(h,g), \tag{5.3}$$

for all $h, g \in C[0,1]$. Define the mapping $V(h(\nu)) = \ln(h(\nu))$. Then the inequality (5.3) can be written as

$$\tau + V\bigl(d(Sh, Tg)\bigr) \leq V(E(h,g)).$$

All the hypotheses of Theorem 2.2 are verified. The mappings S and T admit a unique fixed point, hence Eqs. (5.1) and (5.2) have a unique solution. □

6 Conclusion

In this article, we have given some new results for a pair of fuzzy mappings which are Ciric and Wardowski type contractions. Dominated mappings are used to prove such fixed-point results. Further, results in ordered modular-like spaces involving graphic contractions equipped with graph dominated mappings are presented. The results have been demonstrated graphically by 2D and 3D graphs. This provides justification for our obtained results. In the end, we applied our results to solve electric circuit equations and fractional differential equations.

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