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The zeros of q-shift difference polynomials of meromorphic functions

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Abstract

In this paper, we investigate the value distribution of difference polynomials $f(z)^n f(qz+c)$ and $f^n(z) + a[f(qz+c) - f(z)]$ related to two well-known differential polynomials, where $f(z)$ is a meromorphic function with finite logarithmic order.

MSC: 30D35; 39B12

Keywords: difference equation; meromorphic function; logarithmic order; Nevanlinna theory; difference polynomials

1 Introduction

In this paper, we shall assume that the reader is familiar with the fundamental results and the standard notation of the Nevanlinna value distribution theory of meromorphic functions (see [1, 2]). The term 'meromorphic function' will mean meromorphic in the whole complex plane $\mathbb{C}$. In addition, we will use notations $\rho(f)$ to denote the order of growth of a meromorphic function $f(z)$, $\lambda(f)$ to denote the exponents of convergence of the zero-sequence of a meromorphic function $f(z)$, $\lambda(\frac{1}{f})$ to denote the exponents of convergence of the sequence of distinct poles of $f(z)$.

The non-autonomous Schröder q-difference equation

$$f(qz) = R\bigl(z, f(z)\bigr), \tag{1.1}$$

where the right-hand side is rational in both arguments, has been widely studied during the last decades (see, *e.g.*, [3–7]). There is a variety of methods which can be used to study the value distribution of meromorphic solutions of (1.1).

Recently, the Nevanlinna theory involving q-difference has been developed to study q-difference equations and q-difference polynomials. Many papers have focused on complex difference, giving many difference analogues in value distribution theory of meromorphic functions (see [8–15]).

Hayman [16] posed the following famous conjecture.

Theorem A *If f is a transcendental meromorphic function and $n \geq 1$, then $f^n f'$ takes every finite nonzero value $b \in \mathbb{C}$ infinitely often.*

This conjecture has been solved by Hayman [1] for $n \geq 3$, by Mues [17] for $n = 2$, by Bergweiler and Eremenko [18] for $n = 1$.

Hayman [16] also proved the following famous result.

Theorem B *If $f(z)$ is a transcendental meromorphic function, $n \geq 5$ is an integer, and a $(\neq 0)$ is a constant, then $f'(z) - af(z)^n$ assumes all finite values $b \in \mathbb{C}$ infinitely often.*

He also conjectured in [1] that the same result holds for $n = 3$ and 4. However, Mues [17] proved that the conjecture is not true by providing a counterexample and proved that $f' - af^4$ has infinitely many zeros.

Liu and Qi proved two theorems which considered q-shift difference polynomials (see [13]), which can be seen as difference versions of the above classical results.

Theorem C *Let f be a zero-order transcendental meromorphic function and q be a nonzero complex constant. Then, for $n \geq 6$, $f^n(z)f(qz + c)$ assumes every nonzero value $b \in \mathbb{C}$ infinitely often.*

Theorem D *Let f be a zero-order transcendental meromorphic function and a, q be nonzero complex constants. Then, for $n \geq 8$, $f^n(z) + a[f(qz + c) - f(z)]$ assumes every nonzero value $b \in \mathbb{C}$ infinitely often.*

Remark 1 They also conjectured the numbers $n \geq 6$ and $n \geq 8$ can be reduced in Theorems C and D. But they could not deal with it. In fact, Zhang and Korhonen [19] also proved a result similar to Theorem C under the condition $n \geq 6$. Obviously, it is an interesting question to reduce the number n. In this paper, our results give some answers in some sense.

2 Main results

In order to express our results, we need to introduce some definitions (see [20, 21]).

A positive increasing function $S(r)$, defined for $r > 0$, is said to be of *finite logarithmic order* λ if

$$\limsup_{r\to\infty} \frac{\log S(r)}{\log\log r} = \lambda. \tag{2.1}$$

$S(r)$ is said to be of *infinite logarithmic order* if the limit superior above is infinite.

Definition 1 If $f(z)$ is a function meromorphic in the complex plane $\mathbb{C}$, the logarithmic order of f is the logarithmic order of its characteristic function $T(r,f)$.

It is clear that the logarithmic order of a non-constant rational function is 1, but there exist infinitely many transcendental entire functions of logarithmic order 1 from Theorem 7.3 [20]. Hence, the transcendental meromorphic function is of the logarithmic order ≥ 1.

Let $f(z)$ be a meromorphic function of finite positive logarithmic order λ. A nonnegative continuous function $\lambda(r)$ defined in $(0, +\infty)$ is said to be *proximate logarithmic order* of $T(r,f)$, if $\lambda(r)$ satisfies the following three conditions:

(1) $\lim_{r\to+\infty} \lambda(r) = \lambda$.

(2) $\lambda'(r)$ exists everywhere in $(0, +\infty)$ except possibly in a countable set where $\lambda'(r^+)$ and $\lambda'(r^-)$ exist. Moreover, if we use the one-sided derivative $\lambda'(r^+)$ or $\lambda'(r^-)$ instead of $\lambda'(r)$ of r in the exceptional set, then

$$\lim_{r\to+\infty} r(\log r)\lambda'(r)\log\log r = 0. \tag{2.2}$$

(3) Let $U(r,f) = (\log r)^{\lambda(r)}$, we have $T(r,f) \leq U(r,f)$ for sufficiently large r and

$$\frac{T(r,f)}{U(r,f)} = 1. \tag{2.3}$$

The above function $U(r,f)$ is called a *logarithmic-type function* of $T(r,f)$. If $f(z)$ is a meromorphic function of finite positive logarithmic order λ, then $T(r,f)$ has *proximate logarithmic order* $\lambda(r)$.

Let $f(z)$ be a meromorphic function, for each $a \in \widehat{\mathbb{C}} = \mathbb{C} \cup \{\infty\}$, an a-point of $f(z)$ means a root of the equation $f(z) = a$. Let $\{z_j(a)\}$ be the sequence of a-points of $f(z)$ with $r_j(a) \leq r_{j+1}(a)$, where $r_j(a) = |z_j(a)|$. *The logarithmic exponent of convergence of a-points of* $f(z)$ is a number $\rho_{\log}(a)$ which is defined by

$$\rho_{\log}(a) = \inf\left\{\mu \,\middle|\, \mu > 0, \sum_j 1/\left|\log r_j(a)\right|^{\mu} < +\infty\right\}. \tag{2.4}$$

This quantity plays an important role in measuring the value distribution of a-points of $f(z)$.

Throughout this paper, we denote *the logarithmic order* of $n(r,f = a)$ by $\lambda_{\log}(a)$, where $n(r,f = a)$ is the number of roots of the equation $f(z) = a$ in $|z| \leq r$. It is well known that if a meromorphic function $f(z)$ is of finite order, then the order of $n(r,f = a)$ equals the exponent of convergence of a-points of $f(z)$. The corresponding result for meromorphic functions of finite logarithmic order also holds. That is, if $f(z)$ is a non-constant meromorphic function and of finite logarithmic order, then for each $a \in \widehat{\mathbb{C}}$, the logarithmic order of $n(r,f = a)$ equals the logarithmic exponent of convergence of a-points of $f(z)$.

Although for any given meromorphic function $f(z)$ with finite positive order and for any $a \in \widehat{\mathbb{C}}$, the counting functions $N(r,f = a)$ and $n(r,f = a)$ both have the same order, the situation is different for functions of finite logarithmic order. That is, if $f(z)$ is a non-constant meromorphic function in $\mathbb{C}$, for each $a \in \widehat{\mathbb{C}}$, $N(r,f = a)$ is of logarithmic order $\lambda_{\log}(a) + 1$ where $\lambda_{\log}(a)$ is the logarithmic order of $n(r,f = a)$.

Theorem 2.1 *If $f(z)$ is a transcendental meromorphic function of finite logarithmic order λ, with the logarithmic exponent of convergence of poles less than $\lambda - 1$ and q, c are nonzero complex constants, then for $n \geq 2$, $f^n(z)f(qz + c)$ assumes every value $b \in \mathbb{C}$ infinitely often.*

Remark 2 The following examples show that the hypothesis the logarithmic exponent of convergence of poles $\lambda_{\log}(\infty)$ is less than $\lambda - 1$ is sharp.

Example 1 Let $f(z) = \prod_{j=0}^{\infty}(1 - q^j z)^{-1}$, $0 < |p| < 1$. Then $f(qz) = (1 - z)f(z)$. But $f^n(z)f(qz)$ have only one zero. We know (see [22, 23])

$$T(r,f) = N(r,f) = O\big((\log r)^2\big).$$

Thus, $\lambda = 2$, and the logarithmic exponent of convergence of poles $\lambda_{\log}(\infty)$ is equal to $\lambda - 1 = 1$. Hence, the condition the logarithmic exponent of convergence of poles is less than $\lambda - 1$ cannot be omitted.

Example 2 Let $f(z) = \prod_{j=0}^{\infty}(1 - z/q^j)^{-1}$, $|p| > 1$. Then $f(qz) = \frac{f(z)}{1-qz}$ and $f^n(z)f(qz) = \frac{f(z)^{n+1}}{1-qz}$ have no zeros. Note that $T(r,f) = N(r,f) = O((\log r)^2)$ (see [22–24]), then $\lambda = 2$, and the logarithmic exponent of convergence of poles $\lambda_{\log}(\infty)$ is equal to $\lambda - 1 = 1$. Hence, our condition the logarithmic exponent of convergence of poles is less than $\lambda - 1$ cannot be omitted.

Remark 3 We note that the authors claimed b is nonzero in Theorem C. But b can be zero in Theorem 2.1.

Theorem 2.2 *If $f(z)$ is a transcendental meromorphic function of finite logarithmic order λ, with the logarithmic exponent of convergence of poles less than $\lambda - 1$, and a, q are nonzero complex constants, then for $n \geq 5$, $f^n(z) + a[f(qz + c) - f(z)]$ assumes every value $b \in \mathbb{C}$ infinitely often.*

Remark 4 The authors also claimed b is nonzero in Theorem D. In fact, b can take the zeros in Theorem D from their proof.

In the following, we consider the difference polynomials similar to Theorem 2.2 and Theorem 1.5 in [25].

Theorem 2.3 *If $f(z)$ is a transcendental meromorphic function of finite logarithmic order λ, with the logarithmic exponent of convergence of poles less than $\lambda - 1$, and a, q are nonzero constants, then for $n \geq 3$, $f^n(z) - af(qz + c)$ assumes every value $b \in \mathbb{C}$ infinitely often.*

3 Proof of Theorem 2.1

We need the following lemmas for the proof of Theorem 2.1.

For a transcendental meromorphic function $f(z)$, $T(r,f)$ is usually dominated by three integrated counting functions. However, when $f(z)$ is of finite logarithmic order, $T(r,f)$ can be dominated by two integrated counting functions as the following shows.

Lemma 3.1 ([20]) *If $f(z)$ is a transcendental meromorphic function of finite logarithmic order λ, then for any two distinct extended complex values a and b, we have*

$$T(r,f) \leq N(r,f=a) + N(r,f=b) + o\big(U(r,f)\big), \tag{3.1}$$

where $U(r,f) = (\log r)^{\lambda(r)}$ is a logarithmic-type function of $T(r,f)$. Furthermore, if $T(r,f)$ has a finite lower logarithmic order

$$\mu = \liminf_{r\to+\infty} \frac{\log T(r,f)}{\log\log r}, \tag{3.2}$$

with $\lambda - \mu < 1$, then

$$T(r,f) \leq N(r,f=a) + N(r,f=b) + o\big(T(r,f)\big). \tag{3.3}$$

Lemma 3.2 *If $f(z)$ is a non-constant zero-order meromorphic function and $q \in \mathbb{C}\backslash\{0\}$, then*

$$T\big(r,f(qz)\big) = \big(1 + o(1)\big)T\big(r,f(z)\big) \tag{3.4}$$

on a set of lower logarithmic density 1.

Lemma 3.3 ([26]) *Let $f(z)$ be a meromorphic function of finite order ρ, and let $c \in \mathbb{C}\setminus\{0\}$. Then, for each $\varepsilon > 0$, one has*

$$T\bigl(r, f(z+c)\bigr) = T(r,f) + O\bigl(r^{\rho-1+\varepsilon}\bigr) + O(\log r). \tag{3.5}$$

From the proof of Theorem 2.1 in [26], we know if $f(z)$ is of zero order, then (3.5) can be written into

$$T\bigl(r, f(z+c)\bigr) = T(r,f) + O(\log r). \tag{3.6}$$

From Lemma 3.2 and (3.6), we can obtain

Lemma 3.4 *If $f(z)$ is a non-constant zero-order meromorphic function and $q \in \mathbb{C}\setminus\{0\}$, then*

$$T\bigl(r, f(qz+c)\bigr) = \bigl(1+o(1)\bigr)T\bigl(r, f(z)\bigr) + O(\log r) \tag{3.7}$$

on a set of lower logarithmic density 1.

Remark 5 If $f(z)$ is a transcendental meromorphic function of finite logarithmic order λ, then (3.7) can be rewritten into

$$T\bigl(r, f(qz+c)\bigr) = \bigl(1+o(1)\bigr)T\bigl(r, f(z)\bigr). \tag{3.8}$$

Lemma 3.5 ([27]) *Let f be a non-constant meromorphic function, n be a positive integer. $P(f) = a_n f^n + a_{n-1} f^{n-1} + \cdots + a_1 f$ where a_i is a meromorphic function satisfying $T(r, a_i) = S(r,f)$ $(i = 1, 2, \ldots, n)$. Then*

$$T\bigl(r, P(f)\bigr) = nT(r,f) + S(r,f).$$

Lemma 3.6 *If $f(z)$ is a non-constant zero-order meromorphic function and $q \in \mathbb{C}\setminus\{0\}$, then*

$$N\bigl(r, f(qz+c)\bigr) = \bigl(1+o(1)\bigr)N\bigl(r, f(z)\bigr) + O(\log r) \tag{3.9}$$

on a set of lower logarithmic density 1.

From the proofs of Theorem 1.3 [19] and Theorem 2.2 [26], we can obtain the lemma easily. If $f(z)$ is a transcendental meromorphic function of finite logarithmic order λ, then (3.9) can be rewritten into

$$N\bigl(r, f(qz+c)\bigr) = \bigl(1+o(1)\bigr)N\bigl(r, f(z)\bigr). \tag{3.10}$$

Lemma 3.7 ([13, 28]) *Let $f(z)$ be a non-constant meromorphic function of zero order, and let $q, c \in \mathbb{C}\setminus\{0\}$. Then*

$$m\left(r, \frac{f(qz+c)}{f(z)}\right) = o\bigl\{T(r,f)\bigr\}$$

on a set of logarithmic density 1.

Lemma 3.8 *If $f(z)$ is a transcendental meromorphic function of finite logarithmic order λ and $c \in \mathbb{C}\setminus\{0\}$, and $n \geq 2$ is an integer, set $G(z) = f^n(z)f(qz+c)$, then $T(r,G) = O(T(r,f))$.*

Proof We can rewrite $G(z)$ in the form

$$G(z) = f(z)^{n+1}\frac{f(qz+c)}{f(z)}. \tag{3.11}$$

For each $\varepsilon > 0$, by Lemma 3.7 and (3.11), we get that

$$m(r,G) \leq (n+1)m(r,f) + m\left(r, \frac{f(qz+c)}{f(z)}\right) = (n+1)m(r,f) + o\{T(r,f)\}. \tag{3.12}$$

From Lemma 3.6, we have

$$N(r,G) \leq nN(r,f) + N\bigl(r,f(qz+c)\bigr) \leq \bigl(n+1+o(1)\bigr)N(r,f). \tag{3.13}$$

By (3.12) and (3.13), we have

$$T(r,G) \leq O\bigl(T(r,f)\bigr). \tag{3.14}$$

By Lemma 3.5 and (3.8), we have

$$\begin{aligned} nT\bigl(r,f(z)\bigr) &= T\bigl(r,f^n(z)\bigr) = T\left(r, \frac{G(z)}{f(qz+c)}\right) \leq T(r,G) + T\bigl(r,f(qz+c)\bigr) \\ &= T\bigl(r,G(z)\bigr) + \bigl(1+o(1)\bigr)T\bigl(r,f(z)\bigr). \end{aligned} \tag{3.15}$$

Thus, from (3.15) we have

$$\bigl(n-1-o(1)\bigr)T(r,f) \leq T(r,G). \tag{3.16}$$

That is, $T(r,f) \leq O(T(r,G))$ from $n \geq 2$.

Thus, (3.14) and (3.16) give that $T(r,G) = O(T(r,f))$. □

Proof of Theorem 2.1 Denote $G(z) = f(z)^n f(qz+c)$.

We claim that $G(z)$ is transcendental if $n \geq 2$.

Suppose that $G(z)$ is a rational function $R(z)$. Then $f(z)^n = R(z)/f(qz+c)$. Therefore, by Lemma 3.5 and (3.8), $nT(r,f(z)) = T(r,f(z)^n) = T(r,R(z)/f(qz+c)) \leq T(r,f(qz+c)) + O(\log r) = (1+o(1))T(r,f(z))$, which contradicts $n \geq 2$.

Hence, the claim holds.

By Lemma 3.1, Lemmas 3.5-3.6, Lemma 3.8, and (3.8), we have

$$\begin{aligned} nT\bigl(r,f(z)\bigr) &= T\bigl(r,f(z)^n\bigr) = T\bigl(r,G(z)/f(qz+c)\bigr) \\ &\leq T\bigl(r,F(z)\bigr) + T\bigl(r,f(qz+c)\bigr) + O(1) \\ &= \bigl(1+o(1)\bigr)T\bigl(r,f(z)\bigr) + T(r,G) \\ &= \bigl(1+o(1)\bigr)T\bigl(r,f(z)\bigr) + N(r,G) + N\left(r, \frac{1}{G-b}\right) + o\bigl(U(r,G)\bigr) \end{aligned}$$

$$\begin{aligned} &\leq \bigl(1+o(1)\bigr)T\bigl(r,f(z)\bigr)+\bigl(n+1+o(1)\bigr)N\bigl(r,f(z)\bigr) \\ &\quad + N\left(r,\frac{1}{G-b}\right)+o\bigl(U(r,f)\bigr), \end{aligned} \tag{3.17}$$

where $U(r,f)=(\log r)^{\lambda(r)}$ is a logarithmic-type function of $T(r,f)$.

Note that for the logarithmic exponent of convergence of poles less than $\lambda-1$, we have

$$\limsup_{r\to\infty}\frac{\log N(r,f(z))}{\log\log r}<\lambda. \tag{3.18}$$

Suppose that

$$N\left(r,\frac{1}{G-b}\right)=o\bigl(T(r,G)\bigr). \tag{3.19}$$

By Lemma 3.8, we know (3.19) can be written into

$$N\left(r,\frac{1}{G-b}\right)=o\bigl(T(r,f)\bigr). \tag{3.20}$$

From (3.17) and (3.20), we have

$$\bigl(n-1-o(1)\bigr)T\bigl(r,f(z)\bigr)\leq\bigl(n+1+o(1)\bigr)N\bigl(r,f(z)\bigr)+o\bigl(U(r,f)\bigr) \tag{3.21}$$

for sufficiently large r.

By (3.19), (3.21), and $n\geq 2$, we have

$$\limsup_{r\to\infty}\frac{\log T(r,f)}{\log\log r}<\lambda. \tag{3.22}$$

This contradicts the fact that $T(r,f)$ has logarithmic order λ. Hence,

$$N\left(r,\frac{1}{G-b}\right)\neq o\bigl(T(r,G)\bigr).$$

That is, $G-b$ has infinitely many zeros, then $f^n(z)f(qz+c)-b$ has infinitely many zeros. This completes the proof of Theorem 2.1. □

4 Proof of Theorems 2.2 and 2.3

Let

$$\varphi=\frac{b-af(z)-af(qz+c)}{f^n(z)}. \tag{4.1}$$

By Lemma 3.4 and (3.8), we obtain

$$\begin{aligned} nT\bigl(r,f(z)\bigr) &= T\bigl(r,f(z)^n\bigr)=T\bigl(r,\varphi(z)/\bigl(b-af(z)-af(qz+c)\bigr)\bigr)+O(1) \\ &\leq T\bigl(r,\varphi(z)\bigr)+T\bigl(r,f(qz+c)\bigr)+T\bigl(r,f(z)\bigr)+O(1) \\ &= T\bigl(r,\varphi(z)\bigr)+\bigl(2+o(1)\bigr)T\bigl(r,f(z)\bigr)+O(1), \end{aligned}$$

which implies that

$$T\big(r,\varphi(z)\big) \geq \big(n-2-o(1)\big)T(r,f)+O(1). \tag{4.2}$$

From (4.1), we can easily get

$$T\big(r,\varphi(z)\big) \leq \big(n+2+o(1)\big)T(r,f)+O(1). \tag{4.3}$$

By (4.2)-(4.3) and $n \geq 3$, we have

$$T\big(r,\varphi(z)\big) = O\big(T(r,f)\big). \tag{4.4}$$

We claim that $\varphi(z)$ is transcendental. Suppose that $\varphi(z)$ is rational, it contradicts (4.2) if $n \geq 3$. The claim holds.

Suppose that $N(r,\frac{1}{\varphi-1}) = o\{T(r,\varphi)\}$. In the following, we will get a contradiction. By (4.4), we have

$$N\left(r,\frac{1}{\varphi-1}\right) = o\big\{T(r,f)\big\}. \tag{4.5}$$

Note that for the logarithmic exponent of convergence of poles less than $\lambda-1$, we have

$$\limsup_{r\to\infty}\frac{\log N(r,f(z))}{\log\log r} < \lambda. \tag{4.6}$$

By Lemma 3.1 and (4.4)-(4.5), we obtain

$$\begin{aligned}
T\big(r,\varphi(z)\big) &\leq N\left(r,\frac{1}{\varphi}\right)+N\left(r,\frac{1}{\varphi-1}\right)+o\big(U(r,\varphi)\big)\\
&\leq N\left(r,\frac{1}{b-af(z)-af(qz+c)}\right)+nN(r,f)+N\left(r,\frac{1}{\varphi-1}\right)+o\big(U(r,f)\big)\\
&\leq T\big(r,f(qz)\big)+T\big(r,f(z)\big)+nN(r,f)+N\left(r,\frac{1}{\varphi-1}\right)+o\big(U(r,f)\big)\\
&\leq \big(2+o(1)\big)T\big(r,f(z)\big)+nN(r,f)+N\left(r,\frac{1}{\varphi-1}\right)+o\big(U(r,f)\big)\\
&\leq \big(2+o(1)\big)T\big(r,f(z)\big)+nN(r,f)+o\big(T(r,f)\big)+o\big(U(r,f)\big),
\end{aligned} \tag{4.7}$$

where $U(r,f) = (\log r)^{\lambda(r)}$ is a logarithmic-type function of $T(r,f)$.

By (4.2) and (4.7), we have

$$\big(n-4-o(1)\big)T(r,f) \leq nN(r,f)+o\big(T(r,f)\big)+o\big(U(r,f)\big),$$

for sufficiently large r. Hence, by (4.6) we have

$$\limsup_{r\to\infty}\frac{\log T(r,f)}{\log\log r} < \lambda. \tag{4.8}$$

This contradicts the fact that $T(r,f)$ has logarithmic order λ. Hence,

$$N\left(r,\frac{1}{\varphi-1}\right)\neq o\bigl(T(r,F)\bigr).$$

That is, $\varphi-1$ has infinitely many zeros, then $f^n(z)+a[f(qz+c)-f(z)]-b$ has infinitely many zeros.

This completes the proof of Theorem 2.2.

The proof of Theorem 2.3 is similar to the proof of Theorem 2.2, we omit it here.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors drafted the manuscript, read and approved the final manuscript.

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