Kurbanli *Advances in Difference Equations* 2011, **2011**:40
http://www.advancesindifferenceequations.com/content/2011/1/40

Advances in Difference Equations
a SpringerOpen Journal

RESEARCH **Open Access**

On the behavior of solutions of the system of rational difference equations $x_{n+1} = \frac{x_{n-1}}{y_n x_{n-1} - 1}, \quad y_{n+1} = \frac{y_{n-1}}{x_n y_{n-1} - 1}, \quad z_{n+1} = \frac{1}{y_n z_n}$

Abdullah Selçuk Kurbanli

Correspondence: akurbanli@yahoo.com
Department Of Mathematics, Faculty Of Education, Selcuk University, Konya 42090, Turkey

Abstract

In this article, we investigate the solutions of the system of difference equations $y_{n+1} = \frac{y_{n-1}}{x_n y_{n-1} - 1}$, $y_{n+1} = \frac{y_{n-1}}{x_n y_{n-1} - 1}$, $z_{n+1} = \frac{1}{y_n z_n}$ where x_0, x_{-1}, y_0, y_{-1}, z_0, z_{-1} real numbers such that $y_0 x_{-1} \neq 1$, $x_0 y_{-1} \neq 1$ and $y_0 z_0 \neq 0$.

1. Introduction

In [1], Kurbanli et al. studied the behavior of positive solutions of the system of rational difference equations

$$x_{n+1} = \frac{x_{n-1}}{y_n x_{n-1} + 1}, \; y_{n+1} = \frac{y_{n-1}}{x_n y_{n-1} + 1}.$$

In [2], Cinar studied the solutions of the systems of difference equations

$$x_{n+1} = \frac{1}{y_n}, \; y_{n+1} = \frac{y_n}{x_{n-1} y_{n-1}}.$$

In [3], Kurbanli, studied the behavior of solutions of the system of rational difference equations

$$x_{n+1} = \frac{x_{n-1}}{y_n x_{n-1} - 1}, \; y_{n+1} = \frac{y_{n-1}}{x_n y_{n-1} - 1}, \; z_{n+1} = \frac{z_{n-1}}{y_n z_{n-1} - 1}.$$

In [4], Papaschinnopoulos and Schinas proved the boundedness, persistence, the oscillatory behavior, and the asymptotic behavior of the positive solutions of the system of difference equations

$$x_{n+1} = \sum_{i=0}^{k} A_i / y_{n-i}^{p_i}, \qquad y_{n+1} = \sum_{i=0}^{k} B_i / x_{n-i}^{q_i}$$

In [5], Clark and Kulenović investigate the global stability properties and asymptotic behavior of solutions of the system of difference equations

$$x_{n+1} = \frac{x_n}{a + c y_n}, \qquad y_{n+1} = \frac{y_n}{b + d x_n}.$$

In [6], Camouzis and Papaschinnopoulos studied the global asymptotic behavior of positive solutions of the system of rational difference equations

$$x_{n+1} = 1 + \frac{x_n}{y_{n-m}}, \; y_{n+1} = 1 + \frac{y_n}{x_{n-m}}.$$

In [7], Kulenović and Nurkanović studied the global asymptotic behavior of solutions of the system of difference equations

$$x_{n+1} = \frac{a + x_n}{b + y_n}, \; y_{n+1} = \frac{c + y_n}{d + z_n}, \; z_{n+1} = \frac{e + z_n}{f + x_n}.$$

In [8], Özban studied the positive solutions of the system of rational difference equations

$$x_{n+1} = \frac{1}{y_{n-k}}, \; y_{n+1} = \frac{y_n}{x_{n-m} y_{n-m-k}}.$$

In [9], Zhang et al. investigated the behavior of the positive solutions of the system of the difference equations

$$x_n = A + \frac{1}{y_{n-p}}, \quad y_n = A + \frac{y_{n-1}}{x_{n-r} y_{n-s}}.$$

In [10], Yalcinkaya studied the global asymptotic stability of the system of difference equations

$$z_{n+1} = \frac{t_n z_{n-1} + a}{t_n + z_{n-1}}, \quad t_{n+1} = \frac{z_n t_{n-1} + a}{z_n + t_{n-1}}$$

In [11], Irićanin and Stević studied the positive solutions of the system of difference equations

$$x_{n+1}^{(1)} = \frac{1 + x_n^{(2)}}{x_{n-1}^{(3)}}, \quad x_{n+1}^{(2)} = \frac{1 + x_n^{(3)}}{x_{n-1}^{(4)}}, \; \ldots, \quad x_{n+1}^{(k)} = \frac{1 + x_n^{(1)}}{x_{n-1}^{(2)}},$$

$$x_{n+1}^{(1)} = \frac{1 + x_n^{(2)} + x_{n-1}^{(3)}}{x_{n-2}^{(4)}}, \quad x_{n+1}^{(2)} = \frac{1 + x_n^{(3)} + x_{n-1}^{(4)}}{x_{n-2}^{(5)}}, \; \ldots, \quad x_{n+1}^{(k)} = \frac{1 + x_n^{(1)} + x_{n-1}^{(2)}}{x_{n-2}^{(3)}}$$

Although difference equations are very simple in form, it is extremely difficult to understand throughly the global behavior of their solutions, for example, see Refs. [12-34].

In this article, we investigate the behavior of the solutions of the difference equation system

$$x_{n+1} = \frac{x_{n-1}}{y_n x_{n-1} - 1}, \quad y_{n+1} = \frac{y_{n-1}}{x_n y_{n-1} - 1}, \quad z_{n+1} = \frac{1}{y_n z_n} \tag{1.1}$$

where x_0, x_{-1}, y_0, y_{-1}, z_0, z_{-1} real numbers such that $y_0 x_{-1} \neq 1$, $x_0 y_{-1} \neq 1$ and $y_0 z_0 \neq 0$.

2. Main results

Theorem 1. *Let $y_0 = a$, $y_{-1} = b$, $x_0 = c$, $x_{-1} = d$, $z_0 = e$, $z_{-1} = f$ be real numbers such that $y_0 x_{-1} \neq 1$, $x_0 y_{-1} \neq 1$ and $y_0 z_0 \neq 0$. Let $\{x_n, y_n, z_n\}$ be a solution of the system* (1.1). *Then all solutions of* (1.1) are

$$x_n = \left\{ \frac{d}{(ad-1)^n} \right\}, \quad n --- odd\, c\,(cb-1)^n, \quad n --- even \tag{1.2}$$

$$y_n = \left\{ \frac{b}{(cb-1)^n} \right\}, \ n---odd\, a\,(ad-1)^n, \quad n---even \tag{1.3}$$

$$z_n = \begin{cases} \frac{b^n-1}{a^n e[(ad-1)(cd-1)]^{\sum_{i=1}^{k} i}}, & n---odd \\ \frac{ane(ad-1)^{\sum_{i=1}^{k}(i-1)}(cb-1)^{\sum_{i=1}^{k} i}}{b^n}, & n---even \end{cases} \tag{1.4}$$

Proof. For $n = 0, 1, 2, 3$, we have

$$\begin{aligned} x_1 &= \frac{x_{-1}}{y_0 x_{-1} - 1} = \frac{d}{ad-1}, \\ y_1 &= \frac{y_{-1}}{x_0 y_{-1} - 1} = \frac{b}{cb-1}, \\ z_1 &= \frac{1}{y_0 z_0} = \frac{1}{ae}, \\ x_2 &= \frac{x_0}{y_1 x_0 - 1} = \frac{c}{\frac{b}{cb-1}c - 1} = c(cb-1), \\ y_2 &= \frac{y_0}{x_1 y_0 - 1} = \frac{a}{\frac{d}{ad-1}a - 1} = a(ad-1) \\ z_2 &= \frac{1}{y_1 z_1} = \frac{1}{\frac{b}{cb-1}\frac{1}{ae}} = \frac{(cb-1)ae}{b}, \\ x_3 &= \frac{x_1}{y_2 x_1 - 1} = \frac{\frac{d}{ad-1}}{a\,(ad-1)\frac{d}{ad-1} - 1} = \frac{d}{(ad-1)^2}, \\ y_3 &= \frac{y_1}{x_2 y_1 - 1} = \frac{\frac{b}{cb-1}}{c\,(cb-1)\frac{b}{cb-1} - 1} = \frac{b}{(cb-1)^2}, \\ z_3 &= \frac{1}{y_2 z_2} = \frac{1}{a(ad-1)\frac{(cb-1)ae}{b}} = \frac{b}{a^2 e(ad-1)(cb-1)} \end{aligned}$$

for $n = k$, assume that

$$\begin{aligned} x_{2k-1} &= \frac{x_{2k-3}}{y_{2k-2}x_{2k-3} - 1} = \frac{d}{(ad-1)^k}, \\ x_{2k} &= \frac{x_{2k-2}}{y_{2k-1}x_{2k-2} - 1} = c(cb-1)^k, \\ y_{2k-1} &= \frac{y_{2k-3}}{x_{2k-2}y_{2k-3} - 1} = \frac{b}{(cb-1)^k}, \\ y_{2k} &= \frac{y_{2k-2}}{x_{2k-1}y_{2k-2} - 1} = a(ad-1)^k \end{aligned}$$

and

$$\begin{aligned} z_{2k-1} &= \frac{b^{k-1}}{a^k e[(ad-1)(cb-1)]^{\sum\limits_{i=1}^{k} i}}, \\ z_{2k} &= \frac{a^k e(ad-1)^{\sum\limits_{i=1}^{k}(i-1)}(cb-1)^{\sum\limits_{i=1}^{k} i}}{b^k} \end{aligned}$$

are true. Then, for $n = k + 1$ we will show that (1.2), (1.3), and (1.4) are true. From (1.1), we have

$$x_{2k+1} = \frac{x_{2k-1}}{y_{2k}x_{2k-1} - 1} = \frac{\frac{d}{(ad-1)^k}}{a(ad-1)^k \frac{d}{(ad-1)^k} - 1} = \frac{d}{(ad-1)^{k+1}},$$

$$y_{2k+1} = \frac{y_{2k-1}}{x_{2k}y_{2k-1} - 1} = \frac{\frac{b}{(cb-1)^k}}{c(cb-1)^k \frac{b}{(cb-1)^k} - 1} = \frac{b}{(cb-1)^{k+1}}.$$

Also, similarly from (1.1), we have

$$z_{2k+1} = \frac{1}{y_{2k}z_{2k}} = \frac{1}{a(ad-1)^k \frac{a^k e(ad-1)^{\sum_{i=1}^{k}(i-1)}(cb-1)^{\sum_{i=1}^{k} i}}{b^k}}$$
$$= \frac{b^k}{a^{k+1}e(ad-1)^{\sum_{i=1}^{k} i}(cb-1)^{\sum_{i=1}^{k} i}}.$$

Also, we have

$$x_{2k+2} = \frac{x_{2k}}{y_{2k+1}x_{2k} - 1} = \frac{c(cb-1)^k}{\frac{b}{(cb-1)^{k+1}}c(cb-1)^k - 1} = \frac{c(cb-1)^k}{\frac{b}{(cb-1)}c - 1} = c(cb-1)^{k+1},$$

$$y_{2k+2} = \frac{y_{2k}}{x_{2k+1}y_{2k} - 1} = \frac{a(ad-1)^k}{\frac{d}{(ad-1)^{k+1}}a(ad-1)^k - 1} = \frac{a(ad-1)^k}{\frac{d}{(ad-1)}a - 1} = a(ad-1)^{k+1}$$

and

$$z_{2k+2} = \frac{1}{y_{2k+1}z_{2k+1}} = \frac{1}{\frac{b}{(cb-1)^{k+1}} \frac{b^k}{a^{k+1}e(ad-1)^{\sum_{i=1}^{k} i}(cb-1)^{\sum_{i=1}^{k} i}}}$$
$$= \frac{a^{k+1}e(ad-1)^{\sum_{i=1}^{k} i}(cb-1)^{\sum_{i=1}^{k+1} i}}{b^{k+1}} = \frac{a^{k+1}e(ad-1)^{\sum_{i=1}^{k+1}(i-1)}(cb-1)^{\sum_{i=1}^{k+1} i}}{b^{k+1}}.$$

□

Corollary 1. *Let* $\{x_n, y_n, z_n\}$ *be a solution of the system* (1.1). *Let* a, b, c, d, e, f *be real numbers such that* $ad \neq 1$, $cb \neq 1$, $ae \neq 0$ *and* $b \neq 0$. *Also, if* $ad, cb \in (1, 2)$ *and* $b > a$ *then we have*

$$\lim_{n\to\infty} x_{2n-1} = \lim_{n\to\infty} y_{2n-1} = \lim_{n\to\infty} z_{2n-1} = \infty$$

and

$$\lim_{n\to\infty} x_{2n} = \lim_{n\to\infty} y_{2n} = \lim_{n\to\infty} z_{2n} = 0.$$

Proof. From $ad, cb \in (1, 2)$ and $b > a$ we have $0 < ad - 1 < 1$ and $0 < cb - 1 < 1$. Hence, we obtain

$$\lim_{n\to\infty} x_{2n-1} = \lim_{n\to\infty} \frac{d}{(ad-1)^n} = d \lim_{n\to\infty} \frac{1}{(ad-1)^n} = d. \quad \infty = \begin{cases} -\infty, & d < 0 \\ +\infty, & d > 0 \end{cases},$$

$$\lim_{n\to\infty} y_{2n-1} = \lim_{n\to\infty} \frac{b}{(cb-1)^n} = b \lim_{n\to\infty} \frac{1}{(cb-1)^n} = b. \quad \infty = \begin{cases} -\infty, & b < 0 \\ +\infty, & b > 0 \end{cases}$$

and

$$\lim_{n\to\infty} z_{2n-1} = \lim_{n\to\infty} \frac{b^{n-1}}{a^n e\left[(ad-1)(cb-1)\right]^{\sum_{i=1}^{k} i}} = \frac{1}{e}. \quad \infty = \begin{cases} -\infty, & e < 0 \\ +\infty, & e > 0 \end{cases}$$

Similarly, from ad, $cb \in (1, 2)$ and $b > a$, we have $0 < ad - 1 < 1$ and $0 < cb - 1 < 1$. Hence, we obtain

$$\lim_{n\to\infty} x_{2n} = \lim_{n\to\infty} c(cd-1)^n = c \lim_{n\to\infty} (cd-1)^n = c. \quad 0 = 0,$$
$$\lim_{n\to\infty} y_{2n} = \lim_{n\to\infty} a\,(af-1)^n = a \lim_{n\to\infty} (af-1)^n = a. \quad 0 = 0.$$

and

$$\lim_{n\to\infty} z_{2n} = \lim_{n\to\infty} \frac{a^n e(ad-1)^{\sum_{i=1}^{k}(i-1)}(cb-1)^{\sum_{i=1}^{k} i}}{b^n} = 0.e. \quad 0 = 0.$$

□

Corollary 2. *Let* $\{x_n, y_n, z_n\}$ *be a solution of the system* (1.1). *Let a, b, c, d, e, f be real numbers such that* $ad \neq 1$, $cb \neq 1$, $ae \neq 0$ *and* $b \neq 0$. *If* $a = b$ *and* $cb = ad = 2$ *then we have*

$$\lim_{n\to\infty} x_{2n-1} = d,$$
$$\lim_{n\to\infty} y_{2n-1} = b,$$
$$\lim_{n\to\infty} z_{2n-1} = \frac{1}{ae}$$

and

$$\lim_{n\to\infty} x_{2n} = c,$$
$$\lim_{n\to\infty} y_{2n} = a,$$
$$\lim_{n\to\infty} z_{2n} = e.$$

Proof. From $a = b$ and $cb = ad = 2$ then we have, $cb - 1 = ad - 1 = 1$. Hence, we have

$$\lim_{n\to\infty} (cb-1)^n = 1$$

and

$$\lim_{n\to\infty} (ad-1)^n = 1.$$

Also, we have

$$\lim_{n\to\infty} x_{2n-1} = \lim_{n\to\infty} \frac{d}{(ad-1)^n} = d \lim_{n\to\infty} \frac{1}{(ad-1)^n} = d. \quad 1 = d,$$
$$\lim_{n\to\infty} y_{2n-1} = \lim_{n\to\infty} \frac{b}{(cb-1)^n} = b \lim_{n\to\infty} \frac{1}{(cb-1)^n} = b. \quad 1 = b$$

and

$$\lim_{n\to\infty} z_{2n-1} = \lim_{n\to\infty} \frac{b^{n-1}}{a^n e[(ad-1)(cb-1)]^{\sum_{i=1}^{K} i}} = \lim_{n\to\infty} \frac{1}{ae} \frac{b^{n-1}}{a^{n-1}[(ad-1)(cb-1)]^{\sum_{i=1}^{k} i}} = \frac{1}{ae}.$$

Similarly, we have

$$\lim_{n\to\infty} x_{2n} = \lim_{n\to\infty} c(cb-1)^n = c \lim_{n\to\infty} (cb-1)^n = c. \quad 1 = c,$$
$$\lim_{n\to\infty} y_{2n} = \lim_{n\to\infty} a(ad-1)^n = a \lim_{n\to\infty} (ad-1)^n = a. \quad 1 = a.$$

and

$$\lim_{n\to\infty} z_{2n} = \lim_{n\to\infty} \frac{a^n e(ad-1)^{\sum_{i=1}^{k}(i-1)}(cb-1)^{\sum_{i=1}^{k} i}}{b^n} = 1.\ e = e.$$

□

Corollary 3. *Let* $\{x_n, y_n, z_n\}$ *be a solution of the system* (1.1). *Let a, b, c, d, e, f be real numbers such that* $ad \neq 1$, $cb \neq 1$, $ae \neq 0$ *and* $b \neq 0$. *Also, if* $0 < a, b, c, d, e, f < 1$ *then we have*

$$\lim_{n\to\infty} x_{2n} = \lim_{n\to\infty} y_{2n} = \lim_{n\to\infty} z_{2n} = 0$$

and

$$\lim_{n\to\infty} x_{2n-1} = \lim_{n\to\infty} y_{2n-1} = \lim_{n\to\infty} z_{2n-1} = \infty.$$

Proof. From $0 < a, b, c, d, e, f < 1$ we have $-1 < ad - 1 < 0$ and $-1 < cb - 1 < 0$. Hence, we obtain

$$\lim_{n\to\infty} x_{2n} = \lim_{n\to\infty} c(bc-1)^n = c \lim_{n\to\infty} (bc-1)^n = c. \quad 0 = 0,$$
$$\lim_{n\to\infty} y_{2n} = \lim_{n\to\infty} a(ad-1)^n = a \lim_{n\to\infty} (ad-1)^n = a. \quad 0 = 0$$

and

$$\lim_{n\to\infty} z_{2n} = \lim_{n\to\infty} \frac{a^n e(ad-1)^{\sum_{i=1}^{k}(i-1)}(cb-1)^{\sum_{i=1}^{k} i}}{b^n} = e. \quad 0 = 0.$$

Similarly, we have

$$\lim_{n\to\infty} x_{2n-1} = \lim_{n\to\infty} \frac{d}{(ad-1)^n} = d \lim_{n\to\infty} \frac{1}{(ad-1)^n} = d \lim_{n\to\infty} \frac{1}{(ad-1)^n} = d. \quad \infty = \begin{cases} -\infty, & n-odd \\ +\infty, & n-even \end{cases},$$
$$\lim_{n\to\infty} y_{2n-1} = \lim_{n\to\infty} \frac{b}{(bc-1)^n} = b \lim_{n\to\infty} \frac{1}{(bc-1)^n} = b. \quad \infty = \begin{cases} -\infty, & n-odd \\ +\infty, & n-even \end{cases}.$$

and

$$\lim_{n\to\infty} z_{2n-1} = \lim_{n\to\infty} \frac{b^{n-1}}{a^n e[(ad-1)(cb-1)]^{\sum_{i=1}^{k} i}} = +\infty.$$

□

Corollary 4. *Let* $\{x_n, y_n, z_n\}$ *be a solution of the system* (1.1). *Let a, b, c, d, e, f be real numbers such that* $ad \neq 1$, $cb \neq 1$, $ae \neq 0$, *and* $b \neq 0$. *Also, if* $0 < a, b, c, d, e, f < 1$ *then we have*

$$\lim_{n\to\infty} x_{2n} y_{2n-1} = cb,$$
$$\lim_{n\to\infty} x_{2n-1} y_{2n} = ad$$

and

$$\lim_{n\to\infty} z_{2n-1}z_{2n} = \infty.$$

Proof. The proof is clear from Theorem 1. □

Competing interests
The author declares that they have no competing interests.

Received: 2 March 2011 Accepted: 6 October 2011 Published: 6 October 2011

References
1. Kurbanli, AS, Çinar, C, Yalcinkaya, I: On the behavior of positive solutions of the system of rational difference equations $y_{n+1} = \frac{y_{n-1}}{x_n y_{n-1}+1}, y_{n+1} = \frac{y_{n-1}}{x_n y_{n-1}+1}$. Math Comput Model. **53**(5-6), :1261–1267 (2011). doi:10.1016/j.mcm.2010.12.009
2. Çinar, C: On the positive solutions of the difference equation system $x_{n+1} = \frac{1}{y_n}, y_{n+1} = \frac{y_n}{x_{n-1}y_{n-1}}$. Appl Math Comput. **158**, 303–305 (2004). doi:10.1016/j.amc.2003.08.073
3. Kurbanli, AS: On the behavior of solutions of the system of rational difference equations $x_{n+1} = \frac{x_{n-1}}{y_n x_{n-1}-1}$, $z_{n+1} = \frac{z_{n-1}}{y_n z_{n-1}-1}, z_{n+1} = \frac{z_{n-1}}{y_n z_{n-1}-1}$. Discrete Dynamics Natural and Society **2011**, 12 (2011). Article ID 932362
4. Papaschinopoulos, G, Schinas, CJ: On the system of two difference equations. J Math Anal Appl. **273**, 294–309 (2002). doi:10.1016/S0022-247X(02)00223-8
5. Clark, D, Kulenović, MRS: A coupled system of rational difference equations. Comput Math Appl. **43**, 849–867 (2002). doi:10.1016/S0898-1221(01)00326-1
6. Camouzis, E, Papaschinopoulos, G: Global asymptotic behavior of positive solutions on the system of rational difference equations $x_{n+1} = 1 + \frac{x_n}{y_{n-m}}, y_{n+1} = 1 + \frac{y_n}{x_{n-m}}$. Appl Math Lett. **17**, 733–737 (2004). doi:10.1016/S0893-9659(04)90113-9
7. Kulenović, MRS, Nurkanović, Z: Global behavior of a three-dimensional linear fractional system of difference equations. J Math Anal Appl. **310**, 673–689 (2005)
8. Özban, AY: On the positive solutions of the system of rational difference equations $x_{n+1} = \frac{1}{y_{n-k}}$, $y_{n+1} = \frac{y_n}{x_{n-m}y_{n-m-k}}$. J Math Anal Appl. **323**, 26–32 (2006). doi:10.1016/j.jmaa.2005.10.031
9. Zhang, Y, Yang, X, Megson, GM, Evans, DJ: On the system of rational difference equations $x_n = A + \frac{1}{y_{n-p}}$, $y_n = A + \frac{y_{n-1}}{x_{n-r}y_{n-s}}$. Appl Math Comput. **176**, 403–408 (2006). doi:10.1016/j.amc.2005.09.039
10. Yalcinkaya, I: On the global asymptotic stability of a second-order system of difference equations. Discrete Dyn Nat Soc **2008**, 12 (2008). (Article ID 860152)
11. Iričanin, B, Stević, S: Some systems of nonlinear difference equations of higher order with periodic solutions. Dyn Contin Discrete Impuls Syst Ser A Math Anal. **13**, 499–507 (2006)
12. Agarwal, RP, Li, WT, Pang, PYH: Asymptotic behavior of a class of nonlinear delay difference equations. J Difference Equat Appl. **8**, 719–728 (2002). doi:10.1080/1023619021000000735
13. Agarwal, RP: Difference Equations and Inequalities. Marcel Dekker, New York, 2 (2000)
14. Papaschinopoulos, G, Schinas, CJ: On a system of two nonlinear difference equations. J Math Anal Appl. **219**, 415–426 (1998). doi:10.1006/jmaa.1997.5829
15. Özban, AY: On the system of rational difference equations $x_n = \frac{a}{y_{n-3}}, y_n = \frac{by_{n-3}}{x_{n-q}y_{n-q}}$. Appl Math Comput. **188**, 833–837 (2007). doi:10.1016/j.amc.2006.10.034
16. Clark, D, Kulenovic, MRS, Selgrade, JF: Global asymptotic behavior of a two-dimensional difference equation modelling competition. Nonlinear Anal. **52**, 1765–1776 (2003). doi:10.1016/S0362-546X(02)00294-8
17. Yang, X, Liu, Y, Bai, S: On the system of high order rational difference equations $x_n = \frac{a}{y_{n-p}}, y_n = \frac{by_{n-p}}{x_{n-q}y_{n-q}}$. Appl Math Comput. **171**, 853–856 (2005). doi:10.1016/j.amc.2005.01.092
18. Yang, X: On the system of rational difference equations $x_n = A + \frac{y_{n-1}}{x_{n-p}y_{n-q}}, y_n = A + \frac{x_{n-1}}{x_{n-r}y_{n-s}}$. J Math Anal Appl. **307**, 305–311 (2005). doi:10.1016/j.jmaa.2004.10.045
19. Zhang, Y, Yang, X, Evans, DJ, Zhu, C: On the nonlinear difference equation system $x_{n+1} = A + \frac{y_{n-m}}{x_n}$, $y_{n+1} = A + \frac{x_{n-m}}{y_n}$. Comput Math Appl. **53**, 1561–1566 (2007). doi:10.1016/j.camwa.2006.04.030
20. Yalcinkaya, I, Cinar, C: Global asymptotic stability of two nonlinear difference equations $z_{n+1} = \frac{t_n + z_{n-1}}{t_n z_{n-1} + a}$, $t_{n+1} = \frac{z_n + t_{n-1}}{z_n t_{n-1} + a}$. Fasciculi Mathematici. **43**, 171–180 (2010)
21. Yalcinkaya, I, Çinar, C, Simsek, D: Global asymptotic stability of a system of difference equations. Appl Anal. **87**(6), :689–699 (2008). doi:10.1080/00036810802163279
22. Yalcinkaya, I, Cinar, C: On the solutions of a systems of difference equations. Int J Math Stat Autumn. **9**(A11) (2011)
23. Cinar, C: On the positive solutions of the difference equation $x_{n+1} = \frac{x_{n-1}}{1+x_n x_{n-1}}$. Appl Math Comput. **150**, 21–24 (2004). doi:10.1016/S0096-3003(03)00194-2
24. Cinar, C: On the positive solutions of the difference equation $x_{n+1} = \frac{ax_{n-1}}{1+bx_n x_{n-1}}$. Appl Math Comput. **156**, 587–590 (2004). doi:10.1016/j.amc.2003.08.010
25. Cinar, C: On the positive solutions of the difference equation $x_{n+1} = \frac{x_{n-1}}{1+ax_n x_{n-1}}$. Appl Math Comput. **158**, 809–812 (2004). doi:10.1016/j.amc.2003.08.140
26. Cinar, C: On the periodic cycle of $x(n+1) = \frac{a_n + b_n x_n}{c_n x_{n-1}}$. Appl Math Comput. **150**, 1–4 (2004). doi:10.1016/S0096-3003(03)00182-6

27. Abu-Saris, R, Çinar, C, Yalcinkaya, I: On the asymptotic stability of $x_{n+1} = \frac{a+x_n x_{n-k}}{x_n+x_{n-k}}$. Comput Math Appl. **56**(5), :1172–1175 (2008). doi:10.1016/j.camwa.2008.02.028
28. Çinar, C: On the difference equation $x_{n+1} = \frac{x_{n-1}}{-1+x_n x_{n-1}}$. Appl Math Comput. **158**, 813–816 (2004). doi:10.1016/j.amc.2003.08.122
29. Çinar, C: On the solutions of the difference equation $x_{n+1} = \frac{x_{n-1}}{-1+ax_n x_{n-1}}$. Appl Math Comput. **158**, 793–797 (2004). doi:10.1016/j.amc.2003.08.139
30. Kurbanli, AS: On the behavior of solutions of the system of rational difference equations $x_{n+1} = \frac{x_{n-1}}{y_n x_{n-1}-1}$, $y_{n+1} = \frac{y_{n-1}}{x_n y_{n-1}-1}$. World Appl Sci J. (2010, in press)
31. Elabbasy, EM, El-Metwally, H, Elsayed, EM: On the solutions of a class of difference equations systems. Demonstratio Mathematica. **41**(1), :109–122 (2008)
32. Elsayed, EM: On the solutions of a rational system of difference equations. Fasciculi Mathematici. **45**, 25–36 (2010)
33. Elsayed, EM: Dynamics of a recursive sequence of higher order. Commun Appl Nonlinear Anal. **16**(2), :37–50 (2009)
34. Elsayed, EM: On the solutions of higher order rational system of recursive sequences. Mathematica Balkanica. **21**(3-4), :287–296 (2008)

doi:10.1186/1687-1847-2011-40
Cite this article as: Kurbanli: **On the behavior of solutions of the system of rational difference equations xn +1=xn-1ynxn-1-1,yn+1=yn-1xnyn-1-1,zn+1=1ynzn.** *Advances in Difference Equations* 2011 **2011**:40.