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Research Article

The Permanence and Extinction of a Discrete Predator-Prey System with Time Delay and Feedback Controls

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A discrete predator-prey system with time delay and feedback controls is studied. Sufficient conditions which guarantee the predator and the prey to be permanent are obtained. Moreover, under some suitable conditions, we show that the predator species y will be driven to extinction. The results indicate that one can choose suitable controls to make the species coexistence in a long term.

1. Introduction

The dynamic relationship between predator and its prey has long been and will continue to be one of the dominant themes in both ecology and mathematical ecology due to its universal existence and importance. The traditional predator-prey models have been studied extensively (e.g., see [1–10] and references cited therein), but they are questioned by several biologists. Thus, the Lotka-Volterra type predator-prey model with the Beddington-DeAngelis functional response has been proposed and has been well studied. The model can be expressed as follows:

$$\begin{aligned} x'(t) &= x_1(t)\left(b - a_{11}x(t) - \frac{a_{12}y(t)}{1+\beta x(t)+\gamma y(t)}\right), \\ y'(t) &= y(t)\left(\frac{a_{21}x(t)}{1+\beta x(t)+\gamma y(t)} - d - a_{22}y(t)\right). \end{aligned} \tag{1.1}$$

The functional response in system (1.1) was introduced by Beddington [11] and DeAngelis et al. [12]. It is similar to the well-known Holling type II functional response but has an extra term γy in the denominator which models mutual interference between predators. It can be derived mechanistically from considerations of time utilization [11] or spatial limits on predation. But few scholars pay attention to this model. Hwang [6] showed that the system has no periodic solutions when the positive equilibrium is locally asymptotical stability by using the divergency criterion. Recently, Fan and Kuang [9] further considered the nonautonomous case of system (1.1), that is, they considered the following system:

$$\begin{aligned} x'(t) &= x_1(t)\left(b(t) - a_{11}(t)x(t) - \frac{a_{12}(t)y(t)}{\alpha(t) + \beta(t)x(t) + \gamma(t)y(t)}\right), \\ y'(t) &= y(t)\left(\frac{a_{21}(t)x(t)}{\alpha(t) + \beta(t)x(t) + \gamma(t)y(t)} - d(t)\right). \end{aligned} \tag{1.2}$$

For the general nonautonomous case, they addressed properties such as permanence, extinction, and globally asymptotic stability of the system. For the periodic (almost periodic) case, they established sufficient criteria for the existence, uniqueness, and stability of a positive periodic solution and a boundary periodic solution. At the end of their paper, numerical simulation results that complement their analytical findings were present.

However, we note that ecosystem in the real world is continuously disturbed by unpredictable forces which can result in changes in the biological parameters such as survival rates. Of practical interest in ecosystem is the question of whether an ecosystem can withstand those unpredictable forces which persist for a finite period of time or not. In the language of control variables, we call the disturbance functions as control variables. In 1993, Gopalsamy and Weng [13] introduced a control variable into the delay logistic model and discussed the asymptotic behavior of solution in logistic models with feedback controls, in which the control variables satisfy certain differential equation. In recent years, the population dynamical systems with feedback controls have been studied in many papers, for example, see [13–22] and references cited therein.

It has been found that discrete time models governed by difference equations are more appropriate than the continuous ones when the populations have nonoverlapping generations. Discrete time models can also provide efficient computational models of continuous models for numerical simulations. It is reasonable to study discrete models governed by difference equations. Motivated by the above works, we focus our attention on the permanence and extinction of species for the following nonautonomous predator-prey model with time delay and feedback controls:

$$\begin{aligned} x(n+1) &= x(n)\exp\left(b(n) - a_{11}(n)x(n) - \frac{a_{12}(n)y(n)}{1 + \beta(n)x(n) + \gamma(n)y(n)} + c_1(n)u_1(n)\right), \\ y(n+1) &= y(n)\exp\left(\frac{a_{21}(n)x(n-\tau)}{1 + \beta(n)x(n-\tau) + \gamma(n)y(n-\tau)} - d(n) - a_{22}(n)y(n) - c_2(n)u_2(n)\right), \\ u_1(n+1) &= r(n) - (e_1(n) - 1)u_1(n) - f_1(n)x(n), \\ u_2(n+1) &= (1 - e_2(n))u_2(n) + f_2(n)y(n), \end{aligned} \tag{1.3}$$

where $x(n)$, $y(n)$ are the density of the prey species and the predator species at time n, respectively. $u_i(n)$ ($i = 1,2$) are the feedback control variables. $b(n), a_{11}(n)$ represent the intrinsic growth rate and density-dependent coefficient of the prey at time n, respectively. $d(n), a_{22}(n)$ denote the death rate and density-dependent coefficient of the predator at time n, respectively. $a_{12}(n)$ denotes the capturing rate of the predator; $a_{21}(n)/a_{12}(n)$ represents the rate of conversion of nutrients into the reproduction of the predator. Further, τ is a positive integer.

For the simplicity and convenience of exposition, we introduce the following notations. Let $R_+ = [0, +\infty)$, $Z_+ = \{1, 2, \ldots\}$ and $[k_1, k_2]$ denote the set of integer k satisfying $k_1 \leq k \leq k_2$. We denote $DC_+ : [-\tau, 0] \to R_+$ to be the space of all nonnegative and bounded discrete time functions. In addition, for any bounded sequence $g(n)$, we denote $g^L = \inf_{n \in Z_+} g(n)$, $g^M = \sup_{n \in Z_+} g(n)$.

Given the biological sense, we only consider solutions of system (1.3) with the following initial condition:

$$\begin{aligned}&\left(x(\theta), y(\theta), u_1(\theta), u_2(\theta)\right)\\&\quad = \left(\phi_1(\theta), \phi_2(\theta), \psi_1(\theta), \psi_2(\theta)\right), \quad \phi_i, \psi_i \in DC_+,\ \phi_i(0) > 0,\ \psi_i(0) > 0, \quad i = 1, 2.\end{aligned} \tag{1.4}$$

It is not difficult to see that the solutions of system (1.3) with the above initial condition are well defined for all $n \geq 0$ and satisfy

$$x(n) > 0, \qquad y(n) > 0, \qquad u_i(n) > 0, \qquad n \in Z_+, \quad i = 1, 2. \tag{1.5}$$

The main purpose of this paper is to establish a new general criterion for the permanence and extinction of system (1.3), which is dependent on feedback controls. This paper is organized as follows. In Section 2, we will give some assumptions and useful lemmas. In Section 3, some new sufficient conditions which guarantee the permanence of all positive solutions of system (1.3) are obtained. Moreover, under some suitable conditions, we show that the predator species y will be driven to extinction.

2. Preliminaries

In this section, we present some useful assumptions and state several lemmas which will be useful in the proving of the main results.

Throughout this paper, we will have both of the following assumptions:

(H_1) $r(n)$, $b(n)$, $d(n)$, $\beta(n)$ and $\gamma(n)$ are nonnegative bounded sequences of real numbers defined on Z_+ such that

$$r^L > 0, \qquad b^L \geq 0, \qquad d^L > 0, \tag{2.1}$$

(H_2) $c_i(n), e_i(n), f_i(n)$ and $a_{ij}(n)$ are nonnegative bounded sequences of real numbers defined on Z_+ such that

$$0 < a_{ii}^L < a_{ii}^M < +\infty, \qquad 0 < e_i^L < e_i^M < 1, \quad i = 1, 2. \tag{2.2}$$

Now, we state several lemmas which will be used to prove the main results in this paper.

First, we consider the following nonautonomous equation:

$$x(n+1) = x(n)\exp\big(g(n) - a(n)x(n)\big), \tag{2.3}$$

where functions $a(n)$, $g(n)$ are bounded and continuous defined on Z_+ with a^L, $g^L > 0$. We have the following result which is given in [23].

Lemma 2.1. *Let $x(n)$ be the positive solution of* (2.3) *with $x(0) > 0$, then*

(a) *there exists a positive constant $M > 1$ such that*

$$M^{-1} < \liminf_{n\to\infty} x(n) \le \limsup_{n\to\infty} x(n) \le M \tag{2.4}$$

for any positive solution $x(n)$ of (2.3);

(b) $\lim_{n\to\infty}(x^{(1)}(n) - x^{(2)}(n)) = 0$ *for any two positive solutions $x^{(1)}(n)$ and $x^{(2)}(n)$ of* (2.3).

Second, one considers the following nonautonomous linear equation:

$$\Delta u(n+1) = f(n) - e(n)u(n), \tag{2.5}$$

where functions $f(n)$ and $e(n)$ are bounded and continuous defined on Z_+ with $f^L > 0$ and $0 < e^L \le e^M < 1$. The following Lemma 2.2 is a direct corollary of Theorem 6.2 of L. Wang and M. Q. Wang [24, page 125].

Lemma 2.2. *Let $u(n)$ be the nonnegative solution of* (2.5) *with $u(0) > 0$, then*

(a) $f^L/e^M < \liminf_{n\to\infty} u(n) \le \limsup_{n\to\infty} u(n) \le f^M/e^L$ *for any positive solution $u(n)$ of* (2.5);

(b) $\lim_{n\to\infty}(u^{(1)}(n) - u^{(2)}(n)) = 0$ *for any two positive solutions $u^{(1)}(n)$ and $u^{(2)}(n)$ of* (2.5).

Further, considering the following:

$$\Delta u(n+1) = f(n) - e(n)u(n) + \omega(n), \tag{2.6}$$

where functions $f(n)$ and $e(n)$ are bounded and continuous defined on Z_+ with $f^L > 0$, $0 < e^L \le e^M < 1$ and $\omega(n) \ge 0$. The following Lemma 2.3 is a direct corollary of Lemma 3 of Xu and Teng [25].

Lemma 2.3. *Let $u(n, n_0, u_0)$ be the positive solution of* (2.6) *with $u(0) > 0$, then for any constants $\epsilon > 0$ and $M > 0$, there exist positive constants $\delta(\epsilon)$ and $\widehat{n}(\epsilon, M)$ such that for any $n_0 \in Z_+$ and $|u_0| < M$, when $|\omega(n)| < \delta$, one has*

$$|u(n, n_0, u_0) - u^*(n, n_0, u_0)| < \epsilon \quad \text{for } n > \widehat{n} + n_0, \tag{2.7}$$

where $u^*(n, n_0, u_0)$ is a positive solution of (2.5) *with $u^*(n_0, n_0, u_0) = u_0$.*

Finally, one considers the following nonautonomous linear equation:

$$\Delta u(n+1) = -e(n)u(n) + \omega(n), \tag{2.8}$$

where functions $e(n)$ are bounded and continuous defined on Z_+ with $0 < e^L \leq e^M < 1$ and $\omega(n) \geq 0$. In [25], the following Lemma 2.4 has been proved.

Lemma 2.4. *Let $u(n)$ be the nonnegative solution of* (2.8) *with $u(0) > 0$, then, for any constants $\epsilon > 0$ and $M > 0$, there exist positive constants $\delta(\epsilon)$ and $\hat{n}(\epsilon, M)$ such that for any $n_0 \in Z$ and $|u_0| < M$, when $\omega(n) < \delta$, one has*

$$u(n, n_0, u_0) < \epsilon \quad \text{for } n > \hat{n} + n_0. \tag{2.9}$$

3. Main Results

Theorem 3.1. *Suppose that assumptions (H_1) and (H_2) hold, then there exists a constant $M > 0$ such that*

$$\limsup_{n\to\infty} x(n) < M, \qquad \limsup_{n\to\infty} y(n) < M, \qquad \limsup_{n\to\infty} u_1(n) < M, \qquad \limsup_{n\to\infty} u_2(n) < M, \tag{3.1}$$

for any positive solution $(x(n), y(n), u_1(n), u_2(n))$ of system (1.3).

Proof. Given any solution $(x(n), y(n), u_1(n), u_2(n))$ of system (1.3), we have

$$\Delta u_1(n+1) \leq r(n) - e_1(n)u_1(n), \tag{3.2}$$

for all $n \geq n_0$, where n_0 is the initial time.

Consider the following auxiliary equation:

$$\Delta v(n+1) = r(n) - e_1(n)v(n), \tag{3.3}$$

from assumptions (H_1), (H_2) and Lemma 2.2, there exists a constant $M_1 > 0$ such that

$$\limsup_{n\to\infty} v(n) \leq M_1, \tag{3.4}$$

where $v(n)$ is the solution of (3.3) with initial condition $v(n_0) = u_1(n_0)$. By the comparison theorem, we have

$$u_1(n) \leq v(n), \quad \forall n \geq n_0. \tag{3.5}$$

From this, we further have

$$\limsup_{n\to\infty} u_1(n) \leq M_1. \tag{3.6}$$

Then, we obtain that for any constant $\varepsilon > 0$, there exists a constant $n_1 > n_0$ such that

$$u_1(n) < M_1 + \varepsilon \quad \forall n \geq n_1. \tag{3.7}$$

According to the first equation of system (1.3), we have

$$x(n) \leq x(n) \exp\{b(n) - a_{11}(n)x(n) + c_1(n)(M_1 + \varepsilon)\}, \tag{3.8}$$

for all $n \geq n_1$. Considering the following auxiliary equation:

$$z(n+1) = z(n) \exp\{b(n) - a_{11}(n)z(n) + c_1(n)(M_1 + \varepsilon)\}, \tag{3.9}$$

thus, as a direct corollary of Lemma 2.1, we get that there exists a positive constant $M_2 > 0$ such that

$$\limsup_{n \to \infty} z(n) \leq M_2, \tag{3.10}$$

where $z(n)$ is the solution of (3.9) with initial condition $z(n_1) = x(n_1)$. By the comparison theorem, we have

$$x(n) \leq z(n), \quad \forall n \geq n_1. \tag{3.11}$$

From this, we further have

$$\limsup_{n \to \infty} x(n) \leq M_2. \tag{3.12}$$

Then, we obtain that for any constant $\varepsilon > 0$, there exists a constant $n_2 > n_1$ such that

$$x(n) < M_2 + \varepsilon, \quad \forall n \geq n_2. \tag{3.13}$$

Hence, from the second equation of system (1.3), we obtain

$$y(n+1) \leq y(n) \exp\{a_{21}(n)(M_2 + \varepsilon) - d(n) - a_{22}(n)y(n)\}, \tag{3.14}$$

for all $n \geq n_2 + \tau$. Following a similar argument as above, we get that there exists a positive constant M_3 such that

$$\limsup_{n \to \infty} y(n) < M_3. \tag{3.15}$$

By a similar argument of the above proof, we further obtain

$$\limsup_{n \to \infty} u_2(n) < M_4. \tag{3.16}$$

From (3.6) and (3.12)–(3.16), we can choose the constant $M = \max\{M_1, M_2, M_3, M_4\}$, such that

$$\limsup_{n\to\infty} x(n) < M, \qquad \limsup_{n\to\infty} y(n) < M,$$
$$\limsup_{n\to\infty} u_1(n) < M, \qquad \limsup_{n\to\infty} u_2(n) < M. \tag{3.17}$$

This completes the proof of Theorem 3.1. □

In order to obtain the permanence of system (1.3), we assume that

(H_3) $[b(n) + c_1(n)u_{10}^*(n)]^L > 0$, where $u_{10}^*(n)$ is some positive solution of the following equation:

$$\Delta u(n+1) = r(n) - e_1(n)u(n). \tag{3.18}$$

Theorem 3.2. *Suppose that assumptions* (H_1)–(H_3) *hold, then there exists a constant* $\eta_x > 0$ *such that*

$$\liminf_{n\to\infty} x(n) > \eta_x, \tag{3.19}$$

for any positive solution $(x(n), y(n), u_1(n), u_2(n))$ *of system* (1.3).

Proof. According to assumptions (H_1) and (H_3), we can choose positive constants ε_0 and ε_1 such that

$$\left(b(n) - a_{11}(n)\varepsilon_0 - \frac{a_{12}(n)\varepsilon_1}{1+\gamma(n)\varepsilon_1} + c_1(n)\left(u_{10}^*(n) - \varepsilon_1\right)\right)^L > \varepsilon_0,$$
$$\left(\frac{a_{21}(n)\varepsilon_0}{1+\beta(n)\varepsilon_0} - d(n)\right)^M < -\varepsilon_0. \tag{3.20}$$

Consider the following equation with parameter α_0:

$$\Delta v(n+1) = r(n) - e_1(n)v(n) - f_1(n)\alpha_0. \tag{3.21}$$

Let $u(n)$ be any positive solution of system (3.18) with initial value $u(n_0) = v_0$. By assumptions (H_1)–(H_3) and Lemma 2.2, we obtain that $u(n)$ is globally asymptotically stable and converges to $u_{10}^*(n)$ uniformly for $n \to +\infty$. Further, from Lemma 2.3, we obtain that, for any given $\varepsilon_1 > 0$ and a positive constant $M > 0$ (M is given in Theorem 3.1), there exist constants $\delta_1 = \delta_1(\varepsilon_1) > 0$ and $n_1^* = n_1^*(\varepsilon_1, M) > 0$, such that for any $n_0 \in Z_+$ and $0 \le v_0 \le M$, when $f_1(n)\alpha_0 < \delta_1$, we have

$$\left|v(n, n_0, v_0) - u_{10}^*(n)\right| < \varepsilon_1, \quad \forall n \ge n_0 + n_1^*, \tag{3.22}$$

where $v(n, n_0, v_0)$ is the solution of (3.21) with initial condition $v(n_0, n_0, v_0) = v_0$.

Let $\alpha_0 \leq \min\{\varepsilon_0, \delta_1/(f_1^M + 1)\}$, from (3.20), we obtain that there exist α_0 and n_1 such that

$$b(n) - a_{11}(n)\alpha_0 - \frac{a_{12}(n)\varepsilon_1}{1+\gamma(n)\varepsilon_1} + c_1(n)\left(u_{10}^*(n) - \varepsilon_1\right) > \alpha_0,$$
$$\frac{a_{21}(n)\alpha_0}{1+\beta(n)\alpha_0} - d(n) < -\alpha_0, \qquad f_1(n) < f_1^M + 1, \tag{3.23}$$

for all $n > n_1$.

We first prove that

$$\limsup_{n\to\infty} x(n) \geq \alpha_0, \tag{3.24}$$

for any positive solution $(x(n), y(n), u_1(n), u_2(n))$ of system (1.3). In fact, if (3.24) is not true, then there exists a $\Phi(\theta) = (\phi_1(\theta), \phi_2(\theta), \varphi_1(\theta), \varphi_2(\theta))$ such that

$$\limsup_{n\to\infty} x(n,\Phi) < \alpha_0, \tag{3.25}$$

where $(x(n,\Phi), y(n,\Phi), u_1(n,\Phi), u_2(n,\Phi))$ is the solution of system (1.3) with initial condition $(x(\theta), y(\theta), u_1(\theta), u_2(\theta)) = \Phi(\theta)$, $\theta \in [-\tau, 0]$. So, there exists an $n_2 > n_1$ such that

$$x(n,\Phi) < \alpha_0 \quad \forall n > n_2. \tag{3.26}$$

Hence, (3.26) together with the third equation of system (1.3) lead to

$$\Delta u_1(n+1) > r(n) - e_1(n)u_1(n) - f_1^M\alpha_0, \tag{3.27}$$

for $n > n_2$. Let $v(n)$ be the solution of (3.21) with initial condition $v(n_2) = u_1(n_2)$, by the comparison theorem, we have

$$u_1(n) \geq v(n), \quad \forall n \geq n_2. \tag{3.28}$$

In (3.22), we choose $n_0 = n_2$ and $v_0 = u_1(n_2)$, since $f_1(n)\alpha_0 < \delta_1$, then for given ε_1, we have

$$v(n) = v(n, n_2, u_1(n_2)) > u_{10}^*(n) - \varepsilon_1, \tag{3.29}$$

for all $n \geq n_2 + n_1^*$. Hence, from (3.28), we further have

$$u_1(n) > u_{10}^*(n) - \varepsilon_1, \quad \forall n \geq n_2 + n_1^*. \tag{3.30}$$

From the second equation of system (1.3), we have

$$y(n+1) \leq y(n)\exp\left\{\frac{a_{21}(n)\alpha_0}{1+\beta(n)\alpha_0} - d(n)\right\}, \tag{3.31}$$

for all $n > n_2 + \tau$. Obviously, we have $y(n) \to 0$ as $n \to +\infty$. Therefore, we get that there exists an n_2^* such that

$$y(n) < \varepsilon_1, \tag{3.32}$$

for any $n > n_2 + \tau + n_2^*$. Hence, by (3.26), (3.30), and (3.32), it follows that

$$x(n+1) \geq x(n) \exp\left\{ b(n) - a_{11}(n)\alpha_0 - \frac{a_{12}(n)\varepsilon_1}{1 + \gamma(n)\varepsilon_1} + c_1(n)\left(u_{10}^*(n) - \varepsilon_1\right) \right\}, \tag{3.33}$$

for any $n > n_2 + \tau + \hat{n}^*$, where $\hat{n}^* = \max\{n_1^*, n_2^*\}$. Thus, from (3.23) and (3.33), we have $\lim_{n \to +\infty} x(n) = +\infty$, which leads to a contradiction. Therefore, (3.24) holds.

Now, we prove the conclusion of Theorem 3.2. In fact, if it is not true, then there exists a sequence $\{Z^{(m)}\} = \{(\varphi_1^{(m)}, \varphi_2^{(m)}, \psi_1^{(m)}, \psi_2^{(m)})\}$ of initial functions such that

$$\liminf_{n \to \infty} x\left(n, Z^{(m)}\right) < \frac{\alpha_0}{(m+1)^2}, \quad \forall m = 1, 2, \ldots. \tag{3.34}$$

On the other hand, by (3.24), we have

$$\limsup_{n \to \infty} x\left(n, Z^{(m)}\right) \geq \alpha_0. \tag{3.35}$$

Hence, there are two positive integer sequences $\{s_q^{(m)}\}$ and $\{t_q^{(m)}\}$ satisfying

$$0 < s_1^{(m)} < t_1^{(m)} < s_2^{(m)} < t_2^{(m)} < \cdots < s_q^{(m)} < t_q^{(m)} < \cdots \tag{3.36}$$

and $\lim_{q \to \infty} s_q^{(m)} = \infty$, such that

$$x\left(s_q^{(m)}, Z^{(m)}\right) \geq \frac{\alpha_0}{m+1}, \qquad x\left(t_q^{(m)}, Z^{(m)}\right) \leq \frac{\alpha_0}{(m+1)^2}, \tag{3.37}$$

$$\frac{\alpha_0}{(m+1)^2} \leq x\left(n, Z^{(m)}\right) \leq \frac{\alpha_0}{m+1}, \quad \forall n \in \left[s_q^{(m)} + 1, t_q^{(m)} - 1\right]. \tag{3.38}$$

By Theorem 3.1, for any given positive integer m, there exists a $K^{(m)}$ such that $x(n, Z^{(m)}) < M$, $y(n, Z^{(m)}) < M$, $u_1(n, Z^{(m)}) < M$, and $u_2(n, Z^{(m)}) < M$ for all $n > K^{(m)}$. Because of $s_q^{(m)} \to +\infty$ as $q \to +\infty$, there exists a positive integer $K_1^{(m)}$ such that $s_q^{(m)} > K^{(m)} + \tau$ and $s_q^{(m)} > n_1$ as $q > K_1^{(m)}$. Let $q \geq K_1^{(m)}$, for any $n \in [s_q^{(m)}, t_q^{(m)}]$, we have

$$\begin{aligned} x\left(n+1, Z^{(m)}\right) &\geq x\left(n, Z^{(m)}\right) \exp\left\{ b(n) - a_{11}(n)M - \frac{a_{12}(n)M}{1 + \gamma(n)M} - c_1(n)M \right\} \\ &\geq x\left(n, Z^{(m)}\right) \exp(-\theta_1), \end{aligned} \tag{3.39}$$

where $\theta_1 = \sup_{n\in Z_+}\{b(n) + a_{11}(n)M + a_{12}(n)M/(1+\gamma(n)M) + c_1(n)M\}$. Hence,

$$\begin{aligned}\frac{\alpha_0}{(m+1)^2} \geq x\left(t_q^{(m)}, Z^{(m)}\right) &\geq x\left(s_q^{(m)}, Z^{(m)}\right)\exp\left[-\theta_1\left(t_q^{(m)} - s_q^{(m)}\right)\right] \\ &\geq \frac{\alpha_0}{m+1}\exp\left[-\theta_1\left(t_q^{(m)} - s_q^{(m)}\right)\right].\end{aligned} \tag{3.40}$$

The above inequality implies that

$$t_q^{(m)} - s_q^{(m)} \geq \frac{\ln(m+1)}{\theta_1}, \quad \forall q \geq K_1^{(m)},\ m = 1, 2, \ldots. \tag{3.41}$$

So, we can choose a large enough $\widehat{m}_0$ such that

$$t_q^{(m)} - s_q^{(m)} \geq \widehat{n}^* + \tau + 2, \quad \forall m \geq \widehat{m}_0,\ q \geq K_1^{(m)}. \tag{3.42}$$

From the third equation of system (1.3) and (3.38), we have

$$\begin{aligned}\Delta u_1\left(n+1, Z^{(m)}\right) &\geq r(n) - e_1(n)u_1\left(n, Z^{(m)}\right) - f_1(n)\frac{\alpha_0}{m+1} \\ &\geq r(n) - e_1(n)u_1\left(n, Z^{(m)}\right) - f_1(n)\alpha_0,\end{aligned} \tag{3.43}$$

for any $m \geq \widehat{m}_0$, $q \geq K_1^{(m)}$, and $n \in [s_q^{(m)} + 1, t_q^{(m)}]$. Assume that $v(n)$ is the solution of (3.21) with the initial condition $v(s_q^{(m)} + 1) = u_1(s_q^{(m)} + 1)$, then from comparison theorem and the above inequality, we have

$$u_1\left(n, Z^{(m)}\right) \geq v(n), \quad \forall n \in \left[s_q^{(m)} + 1, t_q^{(m)}\right],\ m \geq \widehat{m}_0,\ q \geq K_1^{(m)}. \tag{3.44}$$

In (3.22), we choose $n_0 = s_q^{(m)} + 1$ and $v_0 = u_1(s_q^{(m)} + 1)$, since $0 < v_0 < M$ and $f_1(n)\alpha_0 < \delta_1$, then for all $n \in [s_q^{(m)} + 1, t_q^{(m)}]$, we have

$$v(n) = v\left(n, s_q^{(m)} + 1, u_1\left(s_q^{(m)} + 1\right)\right) > u_{10}^*(n) - \varepsilon_1, \quad \forall n \in \left[s_q^{(m)} + 1 + \widehat{n}^*, t_q^{(m)}\right]. \tag{3.45}$$

Equation (3.44) together with (3.45) lead to

$$u_1\left(n, Z^{(m)}\right) > u_{10}^*(n) - \varepsilon_1, \tag{3.46}$$

for all $n \in [s_q^{(m)} + 1 + \widehat{n}^*, t_q^{(m)}]$, $q \geq K_1^{(m)}$, and $m \geq \widehat{m}_0$.

From the second equation of system(1.3), we have

$$y(n+1) \le y(n) \exp\left(\frac{a_{21}(n)\alpha_0}{1+\beta(n)\alpha_0} - d(n)\right), \tag{3.47}$$

for $m \ge \widehat{m}_0$, $q \ge K_1^{(m)}$, and $n \in [s_q^{(m)} + \tau, t_q^{(m)}]$. Therefore, we get that

$$y(n) < \varepsilon_1, \tag{3.48}$$

for any $n \in [s_q^{(m)} + \tau + \widehat{n}^*, t_q^{(m)}]$. Further, from the first equation of systems (1.3), (3.46), and (3.48), we obtain

$$\begin{aligned} x\left(n+1, Z^{(m)}\right) &\ge x\left(n, Z^{(m)}\right) \exp\left\{b(n) - a_{11}(n)\alpha_0 - \frac{a_{12}(n)\varepsilon_1}{1+\gamma(n)\varepsilon_1} + c_1(n)\left(u_{10}^*(n) - \varepsilon_1\right)\right\} \\ &\ge x\left(n, Z^{(m)}\right) \exp(\alpha_0), \end{aligned} \tag{3.49}$$

for any $m \ge \widehat{m}_0$, $q \ge K_1^{(m)}$, and $n \in [s_q^{(m)} + 1 + \tau + \widehat{n}^*, t_q^{(m)}]$. Hence,

$$x\left(t_q^{(m)}, Z^{(m)}\right) \ge x\left(t_q^{(m)} - 1, Z^{(m)}\right) \exp(\alpha_0). \tag{3.50}$$

In view of (3.37) and (3.38), we finally have

$$\begin{aligned} \frac{\alpha_0}{(m+1)^2} &\ge x\left(t_q^{(m)}, Z^{(m)}\right) \ge x\left(t_q^{(m)} - 1, Z^{(m)}\right) \exp(\alpha_0) \\ &\ge \frac{\alpha_0}{(m+1)^2} \exp(\alpha_0) > \frac{\alpha_0}{(m+1)^2}, \end{aligned} \tag{3.51}$$

which is a contradiction. Therefore, the conclusion of Theorem 3.2 holds. This completes the proof of Theorem 3.2. □

In order to obtain the permanence of the component $y(n)$ of system (1.3), we next consider the following single-specie system with feedback control:

$$\begin{aligned} x(n+1) &= x(n) \exp\{b(n) - a_{11}(n)x(n) + c_1(n)u_1(n)\}, \\ \Delta u_1(n+1) &= r(n) - e_1(n)u_1(n) - f_1(n)x(n). \end{aligned} \tag{3.52}$$

For system (3.52), we further introduce the following assumption:

(H_4) suppose $\lambda = \max\{|1 - a_{11}^M \overline{x}|, |1 - a_{11}^L \underline{x}|\} + c_1^M < 1$, $\delta = 1 - e_1^L + f_1^M \overline{x} < 1$, where $\overline{x}$, $\underline{x}$ are given in the proof of Lemma 3.3.

For system(3.52), we have the following result.

Lemma 3.3. *Suppose that assumptions* (H_1)–(H_3) *hold, then*

(a) *there exists a constant* $M > 1$ *such that*

$$M^{-1} < \liminf_{n\to\infty} x(n) < \limsup_{n\to\infty} x(n) < M, \qquad \limsup_{n\to\infty} u_1(n) < M, \tag{3.53}$$

for any positive solution $(x(n), u_1(n))$ *of system* (3.52).

(b) *if assumption* (H_4) *holds, then each fixed positive solution* $(x(n), u_1(n))$ *of system* (3.52) *is globally uniformly attractive on* R^2_{+0}.

Proof. Based on assumptions (H_1)–(H_3), conclusion (a) can be proved by a similar argument as in Theorems 3.1 and 3.2.

Here, we prove conclusion (b). Letting $(x^*_{10}(n), u^*_{10}(n))$ be some solution of system (3.52), by conclusion (a), there exist constants $\overline{x}$, $\underline{x}$, and $M > 1$, such that

$$\underline{x} - \varepsilon < x(n), x^*_{10}(n) < \overline{x} + \varepsilon, \qquad u_1(n), u^*_{10}(n) < M, \tag{3.54}$$

for any solution $(x(n), u_1(n))$ of system (3.52) and $n > n^*$. We make transformation $x(n) = x^*_{10}(n)\exp(v_1(n))$ and $u_1(n) = u^*_{10}(n) + v_2(n)$. Hence, system (3.52) is equivalent to

$$\begin{aligned} v_1(n+1) &= \left(1 - a_{11}(n)x^*_{10}(n)\exp\{\theta_1(n)v_1(n)\}\right)v_1(n) + c_1(n)v_2(n), \\ \Delta v_2(n+1) &= -e_1(n)v_2(n) - f_1(n)x^*_{10}(n)\exp\{\theta_2(n)v_1(n)\}v_1(n). \end{aligned} \tag{3.55}$$

According to (H_4), there exists a $\varepsilon > 0$ small enough, such that $\lambda^{\varepsilon} = \max\{|1 - a^M_{11}(\overline{x} + \varepsilon)|, |1 - a^L_{11}(\underline{x} - \varepsilon)|\} + c^M_1 < 1$, $\sigma^{\varepsilon} = 1 - e^L_1 + f^M_1(\overline{x} + \varepsilon) < 1$. Noticing that $\theta_i(n) \in [0,1]$ implies that $x^*_{10}(n)\exp(\theta_i(n)v_1(n))$ $(i = 1,2)$ lie between $x^*_{10}(n)$ and $x(n)$. Therefore, $\underline{x} - \varepsilon < x^*_{10}(n)\exp(\theta_i(n)v_1(n)) < \overline{x} + \varepsilon$, $i = 1,2$. It follows from (3.55) that

$$\begin{aligned} |v_1(n+1)| &\le \left(1 - a_{11}(n)x^*_{10}(n)\exp\{\theta_1(n)v_1(n)\}\right)v_1(n) + c_1(n)v_2(n), \\ |v_2(n+1)| &\le (1 - e_1(n))v_2(n) - f_1(n)x^*_{10}(n)\exp\{\theta_2(n)v_1(n)\}v_1(n). \end{aligned} \tag{3.56}$$

Let $\mu = \max\{\lambda^{\varepsilon}, \sigma^{\varepsilon}\}$, then $0 < \mu < 1$. It follows easily from (3.56) that

$$\max\{|v_1(n+1)|, |v_2(n+1)|\} \le \mu \max\{|v_1(n)|, |v_2(n)|\}. \tag{3.57}$$

Therefore, $\limsup_{n\to\infty} \max\{|v_1(n+1)|, |v_2(n+1)|\} \to 0$, as $n \to +\infty$, and we can easily obtain that $\limsup_{n\to\infty}|v_1(n+1)| = 0$ and $\limsup_{n\to\infty}|v_2(n+1)| = 0$. The proof is completed. □

Considering the following equations:

$$\begin{gathered} x(n+1) = x(n)\exp\{b(n) - a_{11}(n)x(n) - g(n) + c_1(n)u_1(n)\}, \\ \Delta u_1(n+1) = r(n) - e_1(n)u_1(n) - f_1(n)x(n), \end{gathered} \tag{3.58}$$

then we have the following result.

Lemma 3.4. *Suppose that assumptions* (H_1)*–*(H_4) *hold, then there exists a positive constant* δ_2 *such that for any positive solution* $(x(n), u_1(n))$ *of system* (3.58)*, one has*

$$\lim_{n\to\infty}|x(n) - \tilde{x}(n)| = 0, \qquad \lim_{n\to\infty}|u_1(n) - \tilde{u}(n)| = 0, \qquad g(n) \in [0, \delta_2], \tag{3.59}$$

where $(\tilde{x}(n), \tilde{u}(n))$ *is the solution of system* (3.52) *with* $\tilde{x}(n_0) = x(n_0)$ *and* $\tilde{u}(n_0) = u_1(n_0)$*.*

The proof of Lemma 3.4 is similar to Lemma 3.3, one omits it here.

Let $(x^*(n), u_1^*(n))$ be a fixed solution of system (3.52) defined on R^2_{+0}, one assumes that

(H_5) $(-d(n) + (a_{21}(n)x^*(n-\tau)/(1+\beta(n)x^*(n-\tau)))^L > 0$.

Theorem 3.5. *Suppose that assumptions* (H_1)*–*(H_5) *hold, then there exists a constant* $\eta_y > 0$ *such that*

$$\liminf_{n\to\infty} y(n) > \eta_y, \tag{3.60}$$

for any positive solution $(x(n), y(n), u_1(n), u_2(n))$ *of system* (1.3)*.*

Proof. According to assumption (H_5), we can choose positive constants ε_2, ε_3, and n_1, such that for all $n \ge n_1$, we have

$$-d(n) + \frac{a_{21}(n)(x^*(n-\tau) - \varepsilon_3)}{1 + \beta(n)(x^*(n-\tau) - \varepsilon_3) + \gamma(n)\varepsilon_2} - a_{22}(n)\varepsilon_2 - c_2(n)\varepsilon_3 > \varepsilon_2. \tag{3.61}$$

Considering the following equation with parameter α_1:

$$\Delta v(n+1) = -e_2(n)v(n) + f_2(n)\alpha_1, \tag{3.62}$$

by Lemma 2.4, for given $\varepsilon_3 > 0$ and $M > 0$ (M is given in Theorem 3.1.), there exist constants $\delta_3 = \delta_3(\varepsilon_3) > 0$ and $n_3^* = n_3^*(\varepsilon_3, M) > 0$, such that for any $n_0 \in Z_+$ and $0 \le v_0 \le M$, when $f_2(n)\alpha_0 < \delta_3$, we have

$$v(n, n_0, v_0) < \varepsilon_3, \quad \forall n \ge n_0 + n_3^*. \tag{3.63}$$

□

We choose $\alpha_1 < \max\{\varepsilon_2, \delta_3/(1+f_2^M)\}$ if there exists a constant n' such that $a_{12}(n) - \delta_2\gamma(n) \equiv 0$ for all $n > n'$, otherwise $\alpha_1 < \max\{\varepsilon_2, \delta_3/(1+f_2^M), \delta_2/(a_{12}(n) - \delta_2\gamma(n)^M)\}$. Obviously, there exists an $n_2 > n_1$, such that

$$\frac{a_{12}(n)\alpha_1}{1+\gamma(n)\alpha_1} < \delta_2, \quad f_2(n)\alpha_1 < \delta_3, \quad \forall n > n_2. \tag{3.64}$$

Now, We prove that

$$\limsup_{n\to\infty} y(n) \ge \alpha_1, \tag{3.65}$$

for any positive solution $(x(n), y(n), u_1(n), u_2(n))$ of system (1.3). In fact, if (3.65) is not true, then for α_1, there exist a $\Phi(\theta) = (\phi_1(\theta), \phi_2(\theta), \varphi_1(\theta), \varphi_2(\theta))$ and $n_3 > n_2$ such that for all $n > n_3$,

$$y(n,\Phi) < \alpha_1, \tag{3.66}$$

where $\phi_i \in DC_+$ and $\varphi_i \in DC_+$ $(i = 1,2)$. Hence, for all $n > n_3$, one has

$$0 < \frac{a_{12}(n)y(n)}{1+\beta(n)x(n)+\gamma(n)y(n)} < \frac{a_{12}(n)\alpha_1}{1+\gamma(n)\alpha_1} < \delta_2. \tag{3.67}$$

Therefore, from system (1.3), Lemmas 3.3 and 3.4, it follows that

$$\lim_{n\to\infty}|x(n) - x^*(n)| = 0, \qquad \lim_{n\to\infty}|u_1(n) - u^*(n)| = 0, \tag{3.68}$$

for any solution $(x(n), y(n), u_1(n), u_2(n))$ of system (1.3). Therefore, for any small positive constant $\varepsilon_3 > 0$, there exists an n_4^* such that for all $n \ge n_3 + n_4^*$, we have

$$x(n) \ge x_{10}^*(n) - \varepsilon_3. \tag{3.69}$$

From the fourth equation of system (1.3), one has

$$\Delta u_2(n+1) \le -e_2(n)u_2(n) + f_2^M\alpha_1. \tag{3.70}$$

In (3.63), we choose $n_0 = n_3$ and $v_0 = u(n_3)$. Since $f_2(n)\alpha_1 < \delta_3$, then for all $n \ge n_3 + n_3^*$, we have

$$u_2(n) \le \varepsilon_3. \tag{3.71}$$

Equations (3.69), (3.71) together with the second equation of system (1.3) lead to

$$y(n+1) \ge y(n)\exp\left(\frac{a_{21}(n)\left(x_{10}^*(n) - \varepsilon_3\right)}{1+\beta(n)\left(x_{10}^*(n) - \varepsilon_3\right)+\gamma(n)\alpha_1} - d(n) - a_{22}(n)\alpha_1 - c_2(n)\varepsilon_3\right), \tag{3.72}$$

for all $n > n_3 + \tau + \widehat{n}^{**}$, where $\widehat{n}^{**} = \max\{n_3^*, n_4^*\}$. Obviously, we have $y(n) \to +\infty$ as $n \to +\infty$, which is contradictory to the boundedness of solution of system (1.3). Therefore, (3.65) holds.

Now, we prove the conclusion of Theorem 3.5. In fact, if it is not true, then there exists a sequence $Z^{(m)} = \{\phi_1^{(m)}, \phi_2^{(m)}, \psi_1^{(m)}, \psi_2^{(m)}\}$ of initial functions, such that

$$\liminf_{n\to\infty} y\left(n, Z^{(m)}\right) < \frac{\alpha_1}{(m+1)^2}, \quad \forall m = 1, 2, \ldots, \tag{3.73}$$

where $(x(n, Z^{(m)}), y(n, Z^{(m)}), u_1(n, Z^{(m)}), u_2(n, Z^{(m)}))$ is the solution of system (1.3) with initial condition $(x(\theta), y(\theta), u_1(\theta), u_2(\theta)) = Z^{(m)}(\theta)$ for all $\theta \in [-\tau, 0]$. On the other hand, it follows from (3.65) that

$$\limsup_{n\to\infty} y\left(n, Z^{(m)}\right) \geq \alpha_1. \tag{3.74}$$

Hence, there are two positive integer sequences $\{s_q^{(m)}\}$ and $\{t_q^{(m)}\}$ satisfying

$$0 < s_1^{(m)} < t_1^{(m)} < s_2^{(m)} < t_2^{(m)} < \cdots < s_q^{(m)} < t_q^{(m)} < \cdots \tag{3.75}$$

and $\lim_{q\to\infty} s_q^{(m)} = \infty$, such that

$$y\left(s_q^{(m)}, Z^{(m)}\right) \geq \frac{\alpha_1}{m+1}, \qquad y\left(t_q^{(m)}, Z^{(m)}\right) \leq \frac{\alpha_1}{(m+1)^2}, \tag{3.76}$$

$$\frac{\alpha_1}{(m+1)^2} \leq y\left(n, Z^{(m)}\right) \leq \frac{\alpha_1}{m+1}, \quad \forall n \in \left[s_q^{(m)} + 1, t_q^{(m)} - 1\right]. \tag{3.77}$$

By Theorem 3.1, for given positive integer m, there exists a $K^{(m)}$ such that $x(n, Z^{(m)}) < M$, $y(n, Z^{(m)}) < M$, $u_1(n, Z^{(m)}) < M$, and $u_2(n, Z^{(m)}) < M$ for all $n > K^{(m)}$. Because that $s_q^{(m)} \to +\infty$ as $q \to +\infty$, there is a positive integer $K_1^{(m)}$ such that $s_q^{(m)} > K^{(m)} + \tau$ and $s_q^{(m)} > n_2$ as $q > K_1^{(m)}$. Let $q \geq K_1^{(m)}$, for any $n \in [s_q^{(m)}, t_q^{(\mathrm{m})}]$, we have

$$\begin{aligned} y\left(n+1, Z^{(m)}\right) &\geq y\left(n, Z^{(m)}\right) \exp[-d(n) - a_{21}(n)M - a_{22}(n)M - c_2(n)M] \\ &\geq y\left(n, Z^{(m)}\right) \exp(-\theta_2), \end{aligned} \tag{3.78}$$

where $\theta_2 = \sup_{n\in N}\{d(n) + a_{21}(n)M + a_{22}(n)M + c_2(n)M\}$. Hence,

$$\begin{aligned} \frac{\alpha_1}{(m+1)^2} \geq y\left(t_q^{(m)}, Z^{(m)}\right) &\geq y\left(s_q^{(m)}, Z^{(m)}\right) \exp\left[-\theta_2\left(t_{\mathrm{q}}^{(m)} - s_q^{(m)}\right)\right] \\ &\geq \frac{\alpha_1}{m+1} \exp\left[-\theta_2\left(t_q^{(m)} - s_q^{(m)}\right)\right]. \end{aligned} \tag{3.79}$$

The above inequality implies that

$$t_q^{(m)} - s_q^{(m)} \geq \frac{\ln(m+1)}{\theta_2}, \quad \forall q \geq K_1^{(m)},\ m = 1, 2, \ldots. \tag{3.80}$$

Choosing a large enough $\widehat{m}_1$, such that

$$t_q^{(m)} - s_q^{(m)} > \widehat{n}^{**} + \tau + 2, \quad \forall m \geq \widehat{m}_1,\ q \geq K_1^{(m)}, \tag{3.81}$$

then for $m \geq \widehat{m}_1,\ q \geq K_1^{(m)}$, we have

$$0 < \frac{a_{12}(n)y(n)}{1+\beta(n)x(n)+\gamma(n)y(n)} < \frac{a_{12}(n)\alpha_1}{1+\gamma(n)\alpha_1} < \delta_2, \tag{3.82}$$

for all $n \in [s_q^{(m)}+1, t_q^{(m)}]$. Therefore, it follows from system (1.3) that

$$\begin{aligned} x(n+1) &\geq x(n)\exp(b(n) - a_{11}(n)x(n) - \delta_2 + c_1(n)u_1(n)), \\ u_1(n+1) &= r(n) - (e_1(n)-1)u_1(n) - f_1(n)x(n), \end{aligned} \tag{3.83}$$

for all $n \in [s_q^{(m)}+1, t_q^{(m)}]$. Further, by Lemmas 3.3 and 3.4, we obtain that for any small positive constant $\varepsilon_3 > 0$, we have

$$x(n) \geq x_{10}^*(n) - \varepsilon_3, \tag{3.84}$$

for any $m \geq \widehat{m}_1$, $q \geq K_1^{(m)}$, and $n \in [s_q^{(m)} + 1 + n^{**}, t_q^{(m)}]$. For any $m \geq \widehat{m}_1$, $q \geq K_1^{(m)}$, and $n \in [s_q^{(m)}+1, t_q^{(m)}]$, by the first equation of systems (1.3) and (3.77), it follows that

$$\begin{aligned} \Delta u_2\left(n+1, Z^{(m)}\right) &\leq -e_2(n)u_2\left(n, Z^{(m)}\right) + f_2(n)\frac{\alpha_1}{m+1} \\ &\leq -e_2(n)u_2\left(n, Z^{(m)}\right) + f_2(n)\alpha_1. \end{aligned} \tag{3.85}$$

Assume that $v(n)$ is the solution of (3.62) with the initial condition $v(s_q^{(m)}+1) = u_2(s_q^{(m)}+1)$, then from comparison theorem and the above inequality, we have

$$u_2\left(n, Z^{(m)}\right) \leq v(n), \quad \forall n \in \left[s_q^{(m)}+1, t_q^{(m)}\right],\ m \geq \widehat{m}_1,\ q \geq K_1^{(m)}. \tag{3.86}$$

In (3.63), we choose $n_0 = s_q^{(m)}+1$ and $v_0 = u_2(s_q^{(m)}+1)$. Since $0 < v_0 < M$ and $f_2(n)\alpha_1 < \delta_3$, then we have

$$v(n) \leq \varepsilon_3, \quad \forall n \in \left[s_q^{(m)} + 1 + \widehat{n}^{**}, t_q^{(m)}\right]. \tag{3.87}$$

Equation (3.86) together with (3.87) lead to

$$u_2\left(n, \phi^{(m)}, \psi^{(m)}\right) \leq \varepsilon_3, \tag{3.88}$$

for all $n \in [s_q^{(m)} + 1 + \widehat{n}^{**}, t_q^{(m)}]$, $q \geq K_1^{(m)}$, and $m \geq \widehat{m}_1$.

So, for any $m \geq \widehat{m}_1$, $q \geq K_1^{(m)}$, and $n \in [s_q^{(m)} + \tau + 1 + \widehat{n}^{**}, t_q^{(m)}]$, from the second equation of systems (1.3), (3.61), (3.77), (3.84), and (3.88), it follows that

$$\begin{aligned}
y\left(n+1, Z^{(m)}\right) &= y\left(n, Z^{(m)}\right) \exp\left\{-d(n) + \frac{a_{21}(n)x(n-\tau, Z^{(m)})}{1+\beta(n)x(n-\tau, Z^{(m)}) + \gamma(n)y(n-\tau, Z^{(m)})}\right. \\
&\qquad \left. - a_{22}(n)y\left(n, Z^{(m)}\right) - c_2(n)u_2\left(n, Z^{(m)}\right)\right\} \\
&\geq y\left(n, Z^{(m)}\right) \exp\left\{-d(n) + \frac{a_{21}(n)(x_{10}^*(n) - \varepsilon_3)}{1+\beta(n)(x_{10}^*(n) - \varepsilon_3) + \gamma(n)\alpha_1}\right. \\
&\qquad \left. - a_{22}(n)\alpha_1 - c_2(n)\varepsilon_3\right\} \\
&\geq y\left(n, Z^{(m)}\right) \exp\{\alpha_1\}.
\end{aligned} \tag{3.89}$$

Hence,

$$y\left(t_q^{(m)}, Z^{(m)}\right) \geq y\left(t_q^{(m)} - 1, Z^{(m)}\right) \exp(\alpha_1). \tag{3.90}$$

In view of (3.76) and (3.77), we finally have

$$\begin{aligned}
\frac{\alpha_1}{(m+1)^2} &\geq y\left(t_q^{(m)}, Z^{(m)}\right) \geq y\left(t_q^{(m)} - 1, Z^{(m)}\right) \exp(\alpha_1) \\
&\geq \frac{\alpha_1}{(m+1)^2} \exp(\alpha_1) > \frac{\alpha_1}{(m+1)^2},
\end{aligned} \tag{3.91}$$

which is a contradiction. Therefore, the conclusion of Theorem 3.5 holds.

Remark 3.6. In Theorems 3.2 and 3.5, we note that (H_1)–(H_3) are decided by system(1.3), which is dependent on the feedback control $u_1(n)$. So, the control variable $u_1(n)$ has impact on the permanence of system (1.3). That is, there is the permanence of the species as long as feedback controls should be kept beyond the range. If not, we have the following result.

Theorem 3.7. *Suppose that assumption*

$$\left(-d(n) + \frac{a_{21}(n)x^*(n-\tau)}{1+\beta(n)x^*(n-\tau)}\right)^M < 0 \tag{3.92}$$

holds, then

$$\lim_{n\to\infty} y(n) = 0, \tag{3.93}$$

for any positive solution $(x(n), y(n), u_1(n), u_2(n))$ *of system* (1.3).

Proof. By the condition, for any positive constant ε ($\varepsilon < \alpha_1$, where α_1 is given in Theorem 3.5), there exist constants ε_1 and n_1, such that

$$-d(n) + \frac{a_{21}(n)(x^*(n-\tau)+\varepsilon_1)}{1+\beta(n)(x^*(n-\tau)+\varepsilon_1)} - a_{22}(n)\varepsilon < -\varepsilon_1, \tag{3.94}$$

for $n > n_1$. First, we show that there exists an $n_2 > n_1$, such that $y(n_2) < \varepsilon$. Otherwise, there exists an n_1^*, such that

$$y(n) \geq \varepsilon, \quad \forall n > n_1 + n_1^*. \tag{3.95}$$

Hence, for all $n \geq n_1 + n_1^*$, one has

$$\begin{aligned} x(n+1) &< x(n)\exp\left\{b(n) - a_{11}(n)x(n) - \frac{a_{12}(n)\varepsilon}{1+\gamma(n)\varepsilon} + c_1(n)u_1(n)\right\}, \\ \Delta u_1(n+1) &= r(n) - e_1(n)u_1(n) + f_1(n)x(n). \end{aligned} \tag{3.96}$$

Therefore, from Lemma 3.3 and comparison theorem, it follows that for the above ε_1, there exists an $n_2^* > 0$, such that

$$x(n) < x^*(n) + \varepsilon_1, \quad \forall n > n_1 + n_2^*. \tag{3.97}$$

Hence, for $n > n_1 + n_2^*$, we have

$$\begin{aligned} \varepsilon \leq y(n+1) &< y(n)\exp\left\{-d(n) + \frac{a_{21}(n)(x^*(n-\tau)+\varepsilon_1)}{1+\beta(n)(x^*(n-\tau)+\varepsilon_1)} - a_{22}(n)\varepsilon\right\} \\ &\leq y(n_1+n_2^*)\exp\{-\varepsilon_1(n-n_1-n_2^*)\} \longrightarrow 0 \quad \text{as } n \longrightarrow +\infty. \end{aligned} \tag{3.98}$$

So, $\varepsilon < 0$, which is a contradiction. Therefor, there exists an $n_2 > n_1$, such that $y(n_2) < \varepsilon$.

Second, we show that

$$y(n) < \varepsilon \exp\{\mu\}, \quad \forall n > n_2, \tag{3.99}$$

where

$$\mu = \max\nolimits_{n\in Z_+}\left\{d(n) + \frac{a_{21}(n)(x^*(n-\tau)+\varepsilon_1)}{1+\beta(n)(x^*(n-\tau)+\varepsilon_1)} + a_{22}(n)\varepsilon\right\} \tag{3.100}$$

is bounded. Otherwise, there exists an $n_3 > n_2$, such that $y(n_3) \geq \varepsilon \exp\{\mu\}$. Hence, there must exist an $n_4 \in [n_2, n_3 - 1]$ such that $y(n_4) < \varepsilon$, $y(n_4 + 1) \geq \varepsilon$, and $y(n) \geq \varepsilon$ for $n \in [n_4 + 1, n_3]$. Let P_1 be a nonnegative integer, such that

$$n_3 = n_4 + P_1 + 1. \tag{3.101}$$

It follows from (3.101) that

$$\begin{aligned}\varepsilon \exp\{\mu\} &\leq y(n_3) \leq y(n_4) \exp\left\{\sum_{s=n_4}^{n_3-1}\left(-d(s) + \frac{a_{21}(s)(x^*(s-\tau)+\varepsilon_1)}{1+\beta(s)(x^*(s-\tau)+\varepsilon_1)} - a_{22}(s)\varepsilon\right)\right\} \\ &\leq y(n_4) \exp\left\{-d(n_4+P_1) + \frac{a_{21}(n_4+P_1)(x^*(n_4+P_1-\tau)+\varepsilon_1)}{1+\beta(n_4+P_1)(x^*(n_4+P_1-\tau)+\varepsilon_1)} - a_{22}(n_4+P_1)\varepsilon\right\} \\ &< \varepsilon \exp(\mu) \longrightarrow 0,\end{aligned} \tag{3.102}$$

which leads to a contradiction. This shows that (3.99) holds. By the arbitrariness of ε, it immediately follows that $y(n) \to 0$ as $n \to +\infty$. This completes the proof of Theorem 3.7. □

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