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Research Article

Permanence and Stable Periodic Solution for a Discrete Competitive System with Multidelays

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The permanence and the existence of periodic solution for a discrete nonautonomous competitive system with multidelays are considered. Also the stability of the periodic solution is discussed. Numerical examples are given to confirm the theoretical results.

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1. Introduction

In this paper, we consider the permanence and the periodic solution for the following discrete competitive model with M-species and several delays:

$$x_i(n+1) = x_i(n)\exp\left[r_i(n) - \sum_{j=1}^{M}\sum_{l=0}^{m} c_{ij}^{(l)}(n)x_j(n-l)\right], \quad i = 1, 2, \ldots, M. \tag{1.1}$$

In model (1.1), $x_i(n)$ is the population density of the species i at nth time step (year, month, day), $r_i(n)$ represents the intrinsic growth rate of species i at nth time step, and $c_{ij}^{(l)}(n)$ reflects the interspecific or intraspecific competitive intensity of species j to species i with time delay l at nth time step.

As a special case of model (1.1), the following discrete model

$$x(n+1) = x(n)\exp\left[r\left(1 - \frac{x(n)}{K}\right)\right] \tag{1.2}$$

has been investigated as the discrete analogue of the well-known continuous Logistic model [1–4]:

$$\frac{dx}{dt} = rx\left(1 - \frac{x}{K}\right). \tag{1.3}$$

And many complex dynamics, such as periodic cycles and chaotic behavior, were found in model (1.2) [2, 4, 5].

It is well known that the reproduction rate and the carrying capacity are intensively influenced by the environment; therefore the following model with time varying coefficients

$$x(n+1) = x(n)\exp\left[r(n)\left(1 - \frac{x(n)}{K(n)}\right)\right] \tag{1.4}$$

was developed from model (1.2) and has been studied recently in [6].

An equivalent version of model (1.4) can be written as

$$x(n+1) = x(n)\exp[r(n) - c(n)x(n)]. \tag{1.5}$$

Models (1.2), (1.4), and (1.5) are both considering of ecosystems for single-species. As a result of coupling M equations both described by model (1.5), one can write out the following model for M-species:

$$x_i(n+1) = x_i(n)\exp\left[r_i(n) - \sum_{j=1}^{M} c_{ij}(n)x_j(n)\right], \quad i = 1, 2, \ldots, M. \tag{1.6}$$

If $\{c_{ij}(n)\}_{n=0}^{\infty}$ $(i, j = 1, 2, \ldots, M)$ is nonnegative sequences, model (1.6) represents the competitive ecosystem of Lotka-Volterra type with M-species [7]. When $M = 2$, model (1.6) was introduced in [8] and recently has been studied in [9]. The autonomous case of (1.6) when $M = 2$ has been studied in [10] and the following permanent result was obtained [10, Theorem 2].

Lemma 1.1. *If* $r_2a_{11} - r_1a_{21} > 0$, $r_1a_{22} - r_2a_{12} > 0$, *then* (1.6) *is permanent.*

It is well known that the effect of time delay plays an important role in population dynamics [11]; therefore, model (1.1) can be constructed from model (1.6) while considering the effect of time delays. Obviously, models (1.2), (1.4), and (1.5) are special cases of model (1.1) for single-species. Model (1.6) is also special case of model (1.1) without delays. Some aspects of model (1.1) has been discussed in the literature. For example, the global asymptotical stability of (1.1) with $M \geq 2$ and the permanence of (1.1) with $M = 2$ were investigated in [12]. Necessary and sufficient conditions for the permanence of the autonomous case of (1.1) with two-species

$$\begin{aligned} x(n+1) &= x(n)\exp[r_1 - c_{11}x(n-n_1) - c_{12}y(n-n_2)], \\ y(n+1) &= y(n)\exp[r_2 - c_{21}x(n-l_1) - c_{22}y(n-l_2)] \end{aligned} \tag{1.7}$$

were obtained in [13].

In theoretical population dynamics, it is important whether or not all species in multispecies ecosystem can be permanent [14, 15]. Many permanent or persistent results have been obtained for continuous biomathematical models that are governed by differential equation(s). For example, one can refer to [11, 16–21] and references cited therein. However, permanent results on the delayed discrete-time competitive model of Lotka-Volterra type are rarely few [13, 22], especially with M-species ($M > 2$). In this manuscript, first we will obtain new sufficient conditions for the permanence of (1.1) when $M \geq 2$.

The population densities observed in the field are usually oscillatory. What cause such phenomenon is a purpose to model population interactions [9, 23]. We will further investigate the existence and stability of the periodic solution for model (1.1) under the assumption that the coefficients of model (1.1) are all periodic with a common period.

The results obtained in this paper are complements to those related with model (1.1). We give some examples to show that the results here are not enclosed by other earlier works. The paper is organized as follows. In next section, we give some preliminaries and obtain the sufficient conditions which guarantee the permanence of model (1.1). In Section 3, we prove the existence of the positive periodic solution of model (1.1) and obtain the sufficient conditions for the stability of the periodic solution.

2. Preliminaries and Permanence

Due to the biological backgrounds of model (1.1), throughout this paper we make the following basic assumptions.

(H_1) $\{r_i(n)\}_{n=0}^{\infty}$ and $\{c_{ii}^{(0)}(n)\}_{n=0}^{\infty}$ $(i = 1, 2, \ldots, M)$ are sequences bounded from below and from above by positive constants.

(H_2) $\{c_{ij}^{(l)}(n)\}_{n=0}^{\infty}$ $(i, j = 1, 2, \ldots, M, l = 1, 2, \ldots, m)$ and $\{c_{ij}^{(0)}(n)\}_{n=0}^{\infty}$ $(i, j = 1, 2, \ldots, M, i \neq j)$ are nonnegative and bounded sequences.

(H_3) The initial values are given by $x_i(s) = a_i^{(s)} \geq 0, x_i(0) = a_i^{(0)} > 0$ $(i = 1, 2, \ldots, M, s = -m, -m+1, \ldots, -1)$.

Next we give some definitions that will be be used in this paper. We write $\{x(n)\} = \{x_1(n),\ x_2(n), \ldots, x_M(n)\}$ and $\phi(s) = \{a_1^{(s)}, a_2^{(s)}, \ldots, a_M^{(s)}\}$, $s = -m, -m+1, \ldots, -1, 0$.

Definition 2.1. We say that $\{x(n)\}$ is a solution of (1.1) with initial values (H_3) if $\{x(n)\}$ satisfies (1.1) for $n > 0$ and $x(n) = \phi(n), n = -m, -m+1, \ldots, -1, 0$.

Under assumptions (H_1), (H_2), and (H_3), solutions of model (1.1) are all consisting of positive sequences; such solution will be called positive solution of (1.1).

Definition 2.2. Model (1.1) is said to be permanent if there are positive constants m_i and M_i $(i = 1, 2, \ldots, M)$ such that

$$m_i \leq \liminf_{n\to\infty} x_i(n) \leq \limsup_{n\to\infty} x_i(n) \leq M_i, \quad i = 1, 2, \ldots, M \tag{2.1}$$

for each positive solution $\{x(n)\}$ of model (1.1).

Definition 2.3. System (1.1) is strongly persistent if each positive solution $\{x(n)\}$ of (1.1) satisfies

$$\liminf_{n\to\infty} x_i(n) > 0, \quad i = 1, 2, \ldots, M. \tag{2.2}$$

Definition 2.4. If each positive solution $\{x(n)\}$ of model (1.1) satisfies that $\|x(n)-\overline{x}(n)\| \to 0$ as $n \to \infty$, we say that the solution $\{\overline{x}(n)\}$ of (1.1) is globally attractive or globally stable, where $\|\cdot\|$ is the maximum norm of the Banach space R^M.

From Definitions 2.2 and 2.3, we know that model (1.1) is strongly persistent if model (1.1) is permanent. For the sake of simplicity, we introduce the following notations for any sequences $\{y(n)\}_{n=0}^{\infty}$:

$$y^* = \limsup_{n\to\infty} y(n), \qquad y_* = \liminf_{n\to\infty} y(n). \tag{2.3}$$

Next we will discuss the sufficient conditions which guarantee that system (1.1) with initial conditions (H_3) is permanent. In the following, we denote $\{x(n)\}$ as the solutions of system (1.1) with initial conditions (H_3). Clearly, $\{x_i(n)\}_{n=0}^{\infty}$ $(i = 1, 2, \ldots, M)$ is positive sequence.

Lemma 2.5. *If $\{z(k)\}_{k=0}^{\infty}$ satisfies*

$$z(k+1) \le z(k)\exp[r(k) - a(k)z(k)], \tag{2.4}$$

for $k \ge K_1$, where $\{a(k)\}_{k=0}^{\infty}$ is a positive sequence bounded from below and from above by positive constants and K_1 is a positive integer, then there exists positive constant A such that

$$\limsup_{k\to\infty} z(k) \le A, \tag{2.5}$$

and $A = \exp(r^* - 1)/a_*$.

Proof. The proof of this lemma is similar to that of Lemma 1 in [24]; we omit the details. □

Lemma 2.6. *If $\{z(k)\}_{k=0}^{\infty}$ satisfies*

$$z(k+1) \ge z(k)\exp[r(k) - a(k)z(k)], \tag{2.6}$$

for $k \ge K_2$, where $\{a(k)\}_{k=0}^{\infty}$ and $\{r(k)\}_{k=0}^{\infty}$ are positive sequences bounded from below and from above by positive constants, K_2 is a positive integer and $z(K_2) > 0$. Further, assume that $\limsup_{k\to\infty} z(k) \le A$ and $a^*A/r_* > 1$, then

$$\liminf_{k\to\infty} z(k) \ge \frac{r_*}{a^*}\exp\left[r^*\left(1 - \frac{a^*}{r_*}A\right)\right]. \tag{2.7}$$

Proof. The proof of this lemma is similar to that of Lemma 2 in [24]; we omit the details. □

In the following, we denote

$$A_i = \frac{1}{c_{ii*}^{(0)}} \exp\left(r_i^* - 1\right), \quad i = 1, 2, \ldots, M. \tag{2.8}$$

Theorem 2.7. *Assume that*

$$r_{i*} - \sum_{l=1}^{m} c_{ii}^{(l)*} A_i - \sum_{j=1, j \neq i}^{M} \sum_{l=0}^{m} c_{ij}^{(l)*} A_j > 0, \quad i = 1, 2, \ldots, M, \tag{2.9}$$

then model (1.1) *is permanent.*

Proof. From model (1.1), we have

$$x_i(n+1) \leq x_i(n) \exp\left[r_i(n) - c_{ii}^{(0)} x_i(n)\right] \tag{2.10}$$

for $i = 1\ 2, \ldots, M$. Therefore, by Lemma 2.5 there exists positive constants A_i $(i = 1, 2, \ldots, M)$ such that

$$\limsup_{n \to \infty} x_i(n) \leq A_i, \quad i = 1, 2, \ldots, M. \tag{2.11}$$

Hence,

$$x_i(n+1) \geq x_i(n) \exp\left[r_i(n) - \sum_{l=1}^{m} c_{ii}^{(l)} (A_i + \varepsilon) - \sum_{j=1, j \neq i}^{M} \sum_{l=0}^{m} c_{ij}^{(l)} (A_j + \varepsilon) - c_{ii}^{(0)} x_i(n)\right] \tag{2.12}$$

for all large n and $\varepsilon > 0$ $(i = 1, 2, \ldots, M)$. By Lemma 2.6, $\liminf_{n \to \infty} x_i(n) \geq C_i$ $(i = 1, 2, \ldots, M)$ provided that

$$\frac{c_{ii}^{(0)*}}{r_{i*} - \sum_{l=1}^{m} c_{ii}^{(l)} A_i - \sum_{j=1, j \neq i}^{M} \sum_{l=0}^{m} c_{ij}^{(l)} A_j} \frac{1}{c_{ii*}^{(0)}} \exp\left(r_i^* - 1\right) > 1, \tag{2.13}$$

where C_i $(i = 1,\ 2, \ldots, M)$ is a positive constant. Note that $\exp(x - 1) \geq x$ for $x > 0$, then

$$\frac{c_{ii}^{(0)*}}{c_{ii*}^{(0)}} \exp\left(r_i^* - 1\right) \geq r_i^* \geq r_{i*} > r_{i*} - \sum_{l=1}^{m} c_{ii}^{(l)} A_i - \sum_{j=1, j \neq i}^{M} \sum_{l=0}^{m} c_{ij}^{(l)} A_j. \tag{2.14}$$

That is, (2.13) is satisfied if (2.9) holds. Moreover, if (2.9) holds, then $C_i > 0$ and $C_i \leq A_i$ $(i = 1, 2, \ldots, M)$. □

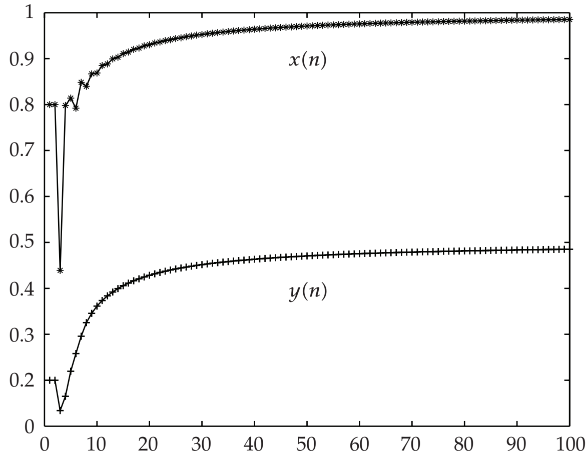

Figure 1: Permanence of model (2.15): the coefficients are given in Example 2.8.

Next we give an example to show the feasibility of the conditions of Theorem 2.7. This example also shows that Theorem 2.7 is not enclosed by other related works.

Example 2.8. Let us consider the following competitive model:

$$x(n+1) = x(n)\exp\Big[r_1 - c_{11}^{(0)}(n)x(n) - c_{11}^{(1)}(n)x(n-1) - c_{12}^{(0)}(n)y(n) - c_{12}^{(1)}(n)y(n-1)\Big],$$
$$y(n+1) = y(n)\exp\Big[r_2 - c_{21}^{(0)}(n)x(n) - c_{21}^{(1)}(n)x(n-1) - c_{22}^{(0)}(n)y(n) - c_{22}^{(1)}(n)y(n-1)\Big], \tag{2.15}$$

where $r_1 = 2, r_2 = 1$, $c_{11}^{(0)}(n) = c_{22}^{(0)}(n) = 2 + 1/n$, $c_{11}^{(1)}(n) = c_{12}^{(0)}(n) = c_{12}^{(1)}(n) = c_{21}^{(0)}(n) = c_{21}^{(1)}(n) = c_{22}^{(1)}(n) = 1/n$.

From (2.15), $c_{11}^{(1)*} = c_{12}^{(0)*} = c_{12}^{(1)*} = c_{21}^{(0)*} = c_{21}^{(1)*} = c_{22}^{(1)*} = 0$, $c_{11}^{(0)*} = c_{22}^{(0)*} = 2$, and hence, (2.9) is satisfied. According to Theorem 2.7, system (2.15) is permanent (see Figure 1).

Remark 2.9. The permanence of system (2.15) was also investigated in [12]. But our conditions which guarantee the permanence of (2.15) are different from that of [12, Lemma 5]. Adopting the same notations as [12, Lemma 5], we have $\overline{r}_1 = 2, \overline{r}_2 = 1$, $\widehat{r}_1 = 2, \widehat{r}_2 = 1$, and $b_{11} = 2$, $b_{12} = 0$, $b_{21} = 0$, $b_{22} = 2$, $B_{11} = 4$, $B_{12} = 2$, $B_{21} = 2$, $B_{22} = 4$, $b_{11}^0 = 2 > 0$, $b_{22}^0 = 2 > 0$. But $\overline{r}_2 b_{11} - \widehat{r}_1 B_{21} = -2$, $\overline{r}_1 b_{22} - \widehat{r}_2 B_{12} = 2$; that is, the assumptions $\overline{r}_2 b_{11} - \widehat{r}_1 B_{21} > 0$ and $\overline{r}_1 b_{22} - \widehat{r}_2 B_{12} > 0$ of [12, Lemma 5] are not satisfied. Therefore, the permanence of system (2.15) cannot be obtained by [12, Lemma 5].

Remark 2.10. The global asymptotical stability of model (1.1) is studied in [12] under the assumption that model (1.1) is strongly persistent. But the authors of [12] did not discuss the strong persistence of model (1.1) with M-species ($M > 2$). Theorem 2.7 in this paper gives sufficient conditions which guarantee the strong persistence of model (1.1).

3. Periodic Solution

In this section, we assume that the coefficients of model (1.1) are periodic with common period ω, that is,

$$r_i(n+\omega) = r_i(n), \quad c_{ij}^{(l)}(n+\omega) = c_{ij}^{(l)}(n), \quad i,j = 1,2,\ldots,M, \quad l = 0,1,2\ldots,m. \tag{3.1}$$

The aim of this section is to show the existence of positive periodic solution of model (1.1) under assumption (3.1) and further find additional conditions for the global stability of this positive periodic solution.

Theorem 3.1. *Let the assumptions of Theorem 2.7 and* (3.1) *be satisfied; then there exists a positive periodic solution of model* (1.1) *with the period ω.*

Proof. Model (1.1) is permanent by Theorem 2.7. Therefore, model (1.1) is point dissipative. It follows from [25, Theorem 4.3] that there exists a positive periodic solution of model (1.1). Note (3.1), and the coefficients of model (1.1) are all ω-periodic. Therefore, this solution is ω-periodic. □

Example 3.2. Consider the following model:

$$\begin{aligned} x(n+1) &= x(n)\exp[1.5 - 2x(n) - 0.01(4 - 3\sin(0.1\pi n))x(n-1) \\ &\qquad -0.01(1+\sin(0.1\pi n))y(n) - 0.001(4+3\cos(0.1\pi n))y(n-1)], \\ y(n+1) &= y(n)\exp[0.8 - 0.01(3+2\sin(0.1\pi n))x(n) - 0.001(1+\cos(0.1\pi n))x(n-1) \\ &\qquad -2y(n) - 0.01(3-2\cos(0.1\pi n))y(n-1)]. \end{aligned} \tag{3.2}$$

Direct computation shows that the coefficients of model (3.2) satisfy the assumptions of Theorem 2.7. The coefficients of model (3.2) are all periodic with the common period 20. Hence, model (3.2) has a positive periodic solution by Theorem 3.1 (see Figures 2 and 3). Figures 4 and 5 show that the period of the sequences $\{x(n)\}$ or $\{y(n)\}$ is 20, respectively. More precisely, the values of sequence x within a period are 0.7330, 0.7358, 0.7383, 0.7403, 0.7415, 0.7419, 0.7413, 0.7399, 0.7377, 0.7351, 0.7323, 0.7295, 0.7271, 0.7252, 0.7241, 0.7238, 0.7243, 0.7257, 0.7277, and 0.7302, respectively. The values of sequence y within a period are 0.3869, 0.3846, 0.3821, 0.3796, 0.3775, 0.3759, 0.3750, 0.3749, 0.3756, 0.3770, 0.3790, 0.3813, 0.3838, 0.3862, 0.3882, 0.3898, 0.3907, 0.3908, 0.3902, and 0.3889, respectively.

Next, we study the global stability of the positive periodic solution obtained in Theorem 3.1.

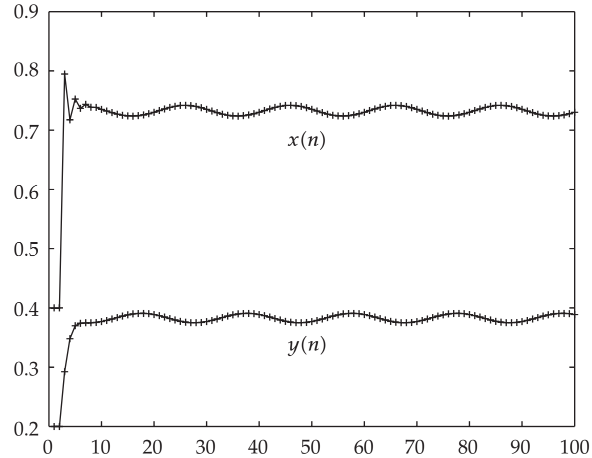

Figure 2: Periodic solution of model (3.2): the coefficients are given in Example 3.2.

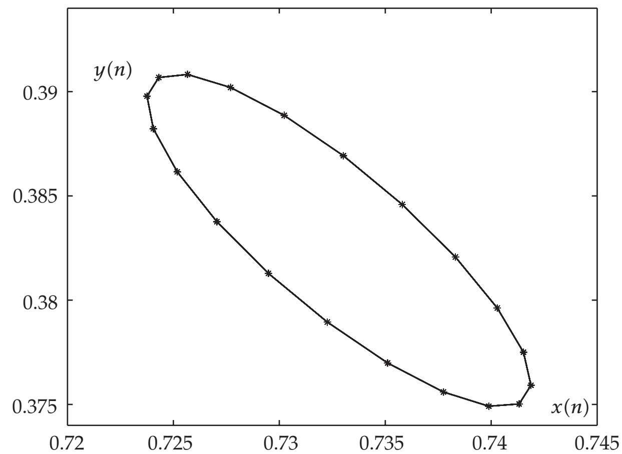

Figure 3: Periodic solution of model (3.2): on the phase plane, the coefficients are given in Example 3.2.

Theorem 3.3. *Let the assumptions of Theorem 3.1 be satisfied; further, assume that*

$$\lambda_i = \max\left\{\left|1 - \sum_{j=1}^{M} c_{ij}^{(0)*} x_i^*\right|, \left|1 - \sum_{j=1}^{M} c_{ij*}^{(0)} x_{i*}\right|\right\} + \sum_{j=1}^{M}\sum_{l=1}^{m} c_{ij}^{(l)*} x_j^* < 1, \tag{3.3}$$

$i = 1, 2, \ldots, M$, *then for every positive solution* $\{x(n)\}$ *of model* (1.1), *one has*

$$\lim_{n\to\infty} |x_i(n) - \overline{x}_i(n)| = 0, \quad i = 1, 2, \ldots, M, \tag{3.4}$$

where $\{\overline{x}(n)\}$ *is the positive periodic solution obtained in Theorem 3.1.*

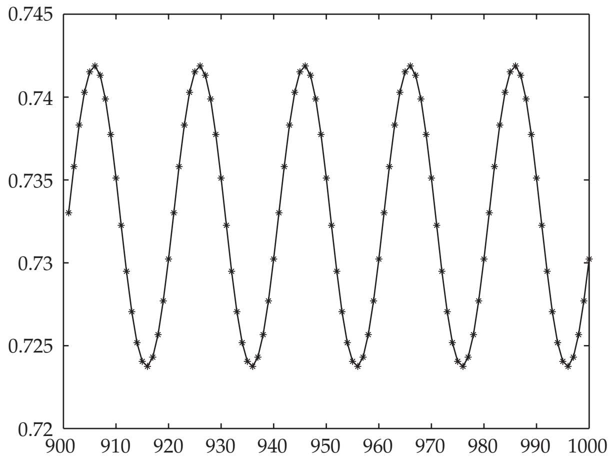

Figure 4: Periodic oscillating of species x: the coefficients are given in Example 3.2.

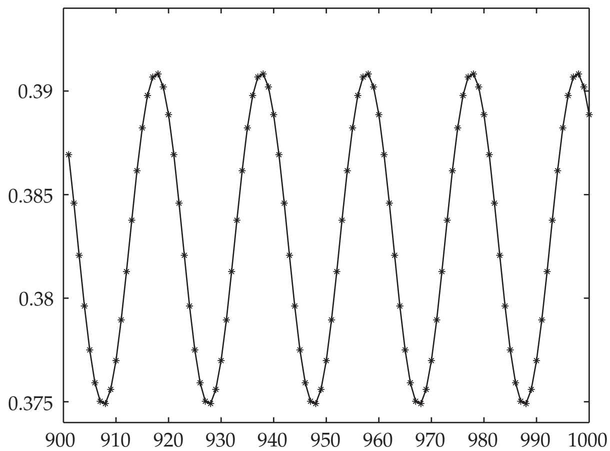

Figure 5: Periodic oscillating of species y: the coefficients are given in Example 3.2.

Proof. Let

$$x_i(n) = \overline{x}_i(n)\exp(u_i(n)), \quad i = 1, 2, \ldots, M, \tag{3.5}$$

where $\overline{x}_i(n) \quad (i = 1, 2, \ldots, M)$ is the positive periodic solution of model (1.1). Model (1.1) can be rewritten as

$$u_i(n+1) = u_i(n) - \sum_{j=1}^{M}\sum_{l=0}^{m} c_{ij}^{(l)}(n)\overline{x}_j(n-l)\left(\exp(u_i(n-l)-1)\right). \tag{3.6}$$

Therefore,

$$u_i(n+1) = u_i(n)\left(1 - \sum_{j=1}^{M} c_{ij}^{(0)}(n)\overline{x}_j(n)\exp(\theta_i(n)u_i(n))\right) - \sum_{j=1}^{M}\sum_{l=1}^{m} c_{ij}^{(l)}(n)\overline{x}_j(n-l)\exp(\nu_i(n-l)u_i(n-l)), \tag{3.7}$$

where $\theta_i(n), \nu_i(n-l) \in [0,1]$, $i = 1,2,\ldots,M$, $l = 1,2,\ldots,m$.

In view of (3.3), we can choose $\varepsilon > 0$ such that

$$\lambda_i^{\varepsilon} = \max\left\{\left|1 - \sum_{j=1}^{M} c_{ij}^{(0)*}(x_i^* + \varepsilon)\right|, \left|1 - \sum_{j=1}^{M} c_{ij*}^{(0)}(x_{i*} - \varepsilon)\right|\right\} + \sum_{j=1}^{M}\sum_{l=1}^{m} c_{ij}^{(l)*}\left(x_j^* + \varepsilon\right) < 1, i = 1,2,\ldots,M. \tag{3.8}$$

And from Theorem 2.7, there exists a positive integer n_0 such that

$$x_{i*} - \varepsilon \le \overline{x}_i(n-l) \le x_i^* + \varepsilon, x_{i*} - \varepsilon \le x_i(n-l) \le x_i^* + \varepsilon \tag{3.9}$$

for $n \ge n_0$ and ε given as above ($i = 1,2,\ldots,M$, $l = 1,2,\ldots,m$).

Notice that $\overline{x}_i(n)\exp(\theta_i(n)u_i(n))$ lies between $\overline{x}_i(n)$ and $x_i(n)$, and $\overline{x}_i(n-l)\exp(\nu_i(n-l)u_i(n-l))$ lies between $\overline{x}_i(n-l)$ and $x_i(n-l)$ ($i = 1,2,\ldots,M$, $l = 1,2,\ldots,m$), from (3.7), we have

$$|u_i(n+1)| \le \max\left\{\left|1 - \sum_{j=1}^{M} c_{ij}^{(0)*}x_i^*\right|, \left|1 - \sum_{j=1}^{M} c_{ij*}^{(0)}x_{i*}\right|\right\}|u_i(n)| + \sum_{j=1}^{M}\sum_{l=1}^{m} c_{ij}^{(l)*}x_i^*|u_i(n-l)|, \quad i = 1,2,\ldots,M \tag{3.10}$$

for $n \ge n_0$ and $i = 1,2,\ldots,M$.

Denote

$$\lambda = \max_{1\le i\le M}\left\{\lambda_i^{\varepsilon}\right\}. \tag{3.11}$$

We have $\lambda < 1$. Notice that ε is arbitrarily given; from (3.10), we get

$$\max_{1\le i\le M}\{|u_i(n+1)|\} \le \lambda \max_{1\le i\le M}\{|u_i(n)|, |u_i(n-1)|, \ldots, |u_i(n-m)|\}, \quad n \ge n_0. \tag{3.12}$$

Therefore,

$$\max_{1\le i\le M}\{|u_i(n)|\} \le \lambda^{n-n_0} \max_{1\le i\le M}\{|u_i(n_0)|, |u_i(n_0-1)|, \dots, |u_i(n_0-m)|\}. \tag{3.13}$$

That is,

$$\lim_{n\to\infty} u_i(n) = 0, \quad i = 1, 2, \dots, M, \tag{3.14}$$

and (3.4) follows consequently. □

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