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Dynamic behaviors of a discrete Lotka-Volterra competitive system with the effect of toxic substances and feedback controls

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Abstract

By noting the fact that the intrinsic growth rate are not positive everywhere, we revisit Lotka-Volterra competitive system with the effect of toxic substances and feedback controls. The corresponding results about permanence and extinction for the species given in (Chen and Chen in Int. J. Biomath. 8(1):1550012, 2015) are extended. Furthermore, a very important fact is found in our results, that is, the feedback controls and toxic substances have no effect on the permanence and extinction of species. Moreover, we also derive sufficient conditions for the global stability of positive solutions. Finally, some numerical simulations show the feasibility of our main results.

MSC: 34D23; 92B05; 34D40

Keywords: feedback controls; discrete; toxic substances; permanence; extinction; global stability

1 Introduction

It is well known that the effect of toxic substances on ecological communities is an important problem, Maynard Smith [2] proposed a model to incorporate the effects of toxic substances in a two-species Lotka-Volterra competitive system by assuming that each of the species produces a substance that is toxic to the other only in the presence of the other species. However, the author did not analyze the model. By constructing a suitable Lyapunov function, Chattopadhyay [3] obtained a set of sufficient conditions which ensure the system admits a unique globally stable positive equilibrium.

Li and Chen [4] generalized the system considered in [2] and [3] to the non-autonomous case:

$$\begin{aligned}
\dot{x}_1(t) &= x_1(t)\bigl[r_1(t) - a_{11}(t)x_1(t) - a_{12}(t)x_2(t) - b_1(t)x_1(t)x_2(t)\bigr],\\
\dot{x}_2(t) &= x_2(t)\bigl[r_2(t) - a_{21}(t)x_1(t) - a_{22}(t)x_2(t) - b_2(t)x_1(t)x_2(t)\bigr],
\end{aligned} \tag{1.1}$$

where $r_i(t)$, $a_{ij}(t)$, $b_i(t)$, $i,j = 1,2$ are assumed to be continuous and bounded above and below by positive constants, $x_1(t)$ and $x_2(t)$ are population density of species x_1 and x_2 at time t, respectively. By using a fluctuation lemma, Li and Chen [4] obtained sufficient conditions which ensure the second species will be driven to extinction while the first one will stabilize at a certain solution of a logistic equation. Their results indicates that toxic substances play an important role in the extinction of species.

It has been found that the discrete time models governed by difference equations are more appropriate than the continuous ones when the size of the population is rarely small or the population has non-overlapping generations [5]. Li and Chen [6] and Huo and Li [7] studied the following discrete model:

$$\begin{aligned} x_1(k+1) &= x_1(k)\exp\bigl\{r_1(k) - a_{11}(k)x_1(k) - a_{12}(k)x_2(k) - b_1(k)x_1(k)x_2(k)\bigr\}, \\ x_2(k+1) &= x_2(k)\exp\bigl\{r_2(k) - a_{21}(k)x_1(k) - a_{22}(k)x_2(k) - b_2(k)x_1(k)x_2(k)\bigr\}. \end{aligned} \tag{1.2}$$

Huo and Li [7] obtained sufficient conditions which ensure the permanence and global stability of the system (1.2). Li and Chen [6] proved that one of the components will be driven to extinction while the other will be globally attractive with any positive solution of a discrete logistic equation under some conditions. Again, their results showed that toxic substances play an important role in the extinction of species.

Based on the work of Li and Chen [6], recently, Chen and Chen [1] proposed a discrete Lotka-Volterra competitive system with the effect of toxic substances and feedback controls:

$$\begin{aligned} x_1(k+1) &= x_1(k)\exp\bigl\{r_1(k) - a_{11}(k)x_1(k) - a_{12}(k)x_2(k) \\ &\qquad - b_1(k)x_1(k)x_2(k) - d_1(k)u_1(k)\bigr\}, \\ x_2(k+1) &= x_2(k)\exp\bigl\{r_2(k) - a_{21}(k)x_1(k) - a_{22}(k)x_2(k) \\ &\qquad - b_2(k)x_1(k)x_2(k) - d_2(k)u_2(k)\bigr\}, \\ u_1(k+1) &= \bigl(1 - e_1(k)\bigr)u_1(k) + f_1(k)x_1(k), \\ u_2(k+1) &= \bigl(1 - e_2(k)\bigr)u_2(k) + f_2(k)x_2(k), \end{aligned} \tag{1.3}$$

where $x_i(k)$ is the density of the ith species at kth generation and $u_i(k)$ is control variable, $i = 1,2$; $r_i(k)$, $a_{ii}(k)$ denote the intrinsic growth rate and density-dependent coefficient of the ith species, respectively, $i = 1,2$. By $b_1(k)$ and $b_2(k)$ are, respectively, shown that each species produces a substance toxic to the other, but only when the other is present. By constructing a discrete Lyapunov type extinction, they found that if assumptions (H_1)-(H_4) in [1] and the following inequalities:

$$\begin{aligned} &\limsup_{k\to\infty} \frac{\sum_{s=k}^{k+w-1} r_2(s)}{\sum_{s=k}^{k+w-1} r_1(s)} < \liminf_{k\to\infty} \frac{b_2(k)}{b_1(k)}, \\ &\liminf_{k\to\infty} \frac{d_2(k)}{e_2(k)} > \limsup_{k\to\infty} \left(\frac{a_{12}(k)}{f_2(k)} \limsup_{k\to\infty} \frac{\sum_{s=k}^{k+w-1} r_2(s)}{\sum_{s=k}^{k+w-1} r_1(s)} - \frac{a_{22}(k)}{f_2(k)} \right), \\ &\limsup_{k\to\infty} \frac{d_1(k)}{e_1(k)} < \liminf_{k\to\infty} \left(\frac{a_{21}(k)}{f_1(k)} \liminf_{k\to\infty} \frac{\sum_{s=k}^{k+w-1} r_1(s)}{\sum_{s=k}^{k+w-1} r_2(s)} - \frac{a_{11}(k)}{f_1(k)} \right), \end{aligned}$$

hold, then we have

$$\lim_{k\to\infty} x_2(k) = 0, \qquad \lim_{t\to\infty} u_2(k) = 0$$

for any positive solution $(x_1(k), x_2(k), u_1(k), u_2(k))$ of system (1.3). They also found that in addition to the conditions of Theorem 3.1 in [1], if $r_1^l > 0$, $d_1^u > 0$ and $f_1^l > 0$ still hold, then the specie x_1 will be permanent while the species x_2 will be driven to extinction. Their results indicate that toxic substances and feedback control variables play an important role in the dynamics of the system. However, they did not consider the permanence of the system and the global stability of positive solutions. In this paper, we extend the corresponding results given in [1] and give the permanence of the system and the global stability of positive solutions. For more work on the dynamic behaviors of the competition system with a toxic substance, one could refer to [1–17] and the references cited therein. For more work on the dynamic behaviors of the feedback control ecosystem, one could refer to [18–29] and the references cited therein.

In [1, 6, 7], the basic assumption is shared that all coefficients are nonnegative. Thus those models may be not completely realistic. If the intrinsic growth rates are not positive everywhere, we need to reconsider the model and will meet some essential difficulties. In this paper we discuss the dynamic behaviors of the competition system (1.3). In Section 2, as preliminaries, some assumptions and lemmas are introduced. In Section 3, we establish sufficient conditions on the permanence for system (1.3). In Section 4, we show the global stability of the system (1.3). In Section 5, some sufficient conditions for the extinction of the system (1.3) are obtained. In Section 6, a numerical simulation is presented to illustrate the feasibility of our main result.

2 Preliminaries

For any bounded sequence $x(k)$, we denote $x^u = \sup_{k\in Z}\{x(k)\}$, $x^l = \inf_{k\in Z}\{x(k)\}$, where $Z = \{0, 1, 2, 3, \dots\}$. Throughout this paper, we introduce the following assumptions.

(H$_1$) $r_i(k)$ is a bounded sequence defined on Z; $e_i(k)$ is a positive bounded sequence defined on Z; $a_{ij}(k)$, $b_i(k)$, $d_i(k)$ and $f_i(k)$, $i, j = 1, 2$ are nonnegative bounded sequences defined on Z.

(H$_2$) Sequences $e_i(k)$, $i = 1, 2$ satisfy $0 < e_i^l \leq e_i^u < 1$ for all $k \in Z$.

(H$_3$) There exist positive integers λ_i such that

$$\liminf_{k\to\infty} \sum_{s=k}^{k+\lambda_i-1} a_{ii}(s) \geq 0, \quad i = 1, 2.$$

(H$_4$) There exist positive integers ω_i such that

$$\limsup_{k\to\infty} \sum_{s=k}^{k+\omega_i-1} r_i(s) \leq 0, \quad i = 1, 2.$$

Motivated by the biological background of system (1.3), in this paper we only consider all solutions of system (1.3) that satisfy the initial conditions $x_i(0) > 0$, $u_i(0) > 0$, $i = 1, 2$. It is obvious that the solution $(x_1(k), x_2(k), u_1(k), u_2(k))$ is positive, that is, $x_i(k) > 0$, $u_i(k) > 0$, $i = 1, 2$ for all $k \in Z$.

We consider the following non-autonomous difference inequality system:

$$x(k+1) \leq x(k)\exp\bigl\{a(k) - b(k)x(k)\bigr\}, \tag{2.1}$$

where $a(k)$ and $b(k)$ are bounded sequences and $b(k) \geq 0$ for all $k \in Z$. We get the following result.

Lemma 2.1 ([28]) *Assume that there exist an integer* $\lambda > 0$ *such that*

$$\liminf_{k\to\infty} \sum_{s=k}^{k+\lambda-1} b(s) > 0.$$

Then there exists a constant $M > 0$ *such that, for any nonnegative solution* $x(k)$ *of system* (2.1) *with initial value* $x(k_0) = x_0 \geq 0$, *where* $k_0 \in Z$ *is some integer,*

$$\limsup_{k\to+\infty} x(k) < M.$$

Next, we consider the following non-autonomous linear difference equation:

$$\nu(k+1) \leq \gamma(k)\nu(k) + \omega(k), \tag{2.2}$$

where $\gamma(k)$ and $\omega(k)$ are nonnegative bounded sequences defined on Z. We have the following results.

Lemma 2.2 ([28]) *Assume that there exist an integer* $\lambda > 0$ *such that*

$$\limsup_{k\to\infty} \prod_{s=k}^{k+\lambda-1} \gamma(s) < 1,$$

then there exists a constant $M > 0$ *such that, for any nonnegative solution* $\nu(k)$ *of system* (2.2) *with initial value* $\nu(k_0) = \nu_0 \geq 0$, *where* $k_0 \in Z$ *is some integer,*

$$\limsup_{k\to\infty} \nu(k) < M.$$

Lemma 2.3 ([28]) *Assume that the conditions of Lemma* 2.2 *hold, then for any constants* $\varepsilon > 0$ *and* $M_1 > 0$ *there exist positive constants* $\hat{\delta} = \hat{\delta}(\varepsilon)$ *and* $\hat{k} = \hat{k}(\varepsilon, M_1)$ *such that, for any* $\hat{k}_0 \in Z$ *and* $0 \leq \nu_0 \leq M_1$, *where* $\omega(k) < \hat{\delta}$ *for all* $k \geq \hat{k}_0$, *one has*

$$\nu(k, \hat{k}_0, \nu_0) < \varepsilon \quad \text{for all } k \geq \hat{k}_0 + \hat{k},$$

where $\nu(k, \hat{k}_0, \nu_0)$ *is the solution of* (2.2) *with initial value* $\nu(\hat{k}_0) = \nu_0$.

Lemma 2.4 ([29]) *Assume that* $A > 0$ *and* $y(0) > 0$. *Suppose that*

$$y(k+1) \geq Ay(k) + B(k), \quad k \in N.$$

If $A < 1$ *and* B *is bounded above with respect to* N, *then*

$$\liminf_{k\to+\infty} y(k) \geq \frac{N}{1-A}.$$

3 Permanence

Theorem 3.1 *Assume that assumptions* (H_1)-(H_3) *hold, then there exist constants* $\bar{x}_i, \bar{u}_i > 0$ *such that*

$$\limsup_{k\to\infty} x_i(k) < \bar{x}_i, \qquad \limsup_{n\to\infty} u_i(k) < \bar{u}_i, \quad i = 1,2$$

for any positive solution $(x_1(k), x_2(k), u_1(k), u_2(k))$ *of system* (1.3).

Proof From the first and second equation of system (1.3), we have

$$x_i(k+1) \leq x_i(k)\exp\bigl\{r_i(k) - a_{ii}(k)x_i(k)\bigr\}, \tag{3.1}$$

then by assumption (H_3) and applying Lemma 2.1 there exist constants $\bar{x}_i > 0$ such that

$$\limsup_{k\to\infty} x_i(k) < \bar{x}_i, \quad i = 1,2. \tag{3.2}$$

Hence, there exists a positive integer k_1 such that

$$x_i(k) \leq \bar{x}_i \quad \text{for all } k \geq k_1, i = 1,2.$$

Thus, from the third and fourth equation of system (1.3), we obtain

$$u_i(k+1) \leq \bigl(1 - e_i(k)\bigr)u_i(k) + f_i(k)\bar{x}_i \quad \text{for all } k \geq k_1. \tag{3.3}$$

By assumption (H_2) we can find that there exists a positive integer ρ such that for $i = 1,2$

$$\limsup_{k\to\infty} \prod_{s=k}^{k+\rho-1} \bigl(1 - e_i(s)\bigr) < 1.$$

It follows from Lemma 2.2 that there exist positive constants $\bar{u}_i$ such that

$$\limsup_{k\to\infty} u_i(k) < \bar{u}_i, \quad i = 1,2. \tag{3.4}$$

The proof of Theorem 3.1 is completed. □

In order to obtain the permanence of system (1.3), we assume the following.

(H_5) There exists a positive integer ω_i such that

$$\liminf_{k\to\infty} \sum_{s=k}^{k+\omega_i-1} \bigl(r_i(s) - a_{i3-i}(s)\bar{x}_{3-i}\bigr) > 0, \quad i = 1,2.$$

Theorem 3.2 *Suppose that* (H_1)-(H_3) *and* (H_5) *hold, then the system of* (1.3) *is permanent.*

Proof From Theorem 3.1, it follows that there exist constants $\bar{x}_i, \bar{u}_i > 0$ such that

$$\limsup_{k\to\infty} x_i(k) < \bar{x}_i, \qquad \limsup_{n\to\infty} u_i(k) < \bar{u}_i, \quad i = 1,2$$

for any positive solution $(x_1(k), x_2(k), u_1(k), u_2(k))$ of system (1.3).

Next, we can only prove that there exist constants $\underline{x}_i, \underline{u}_i > 0$ such that

$$\liminf_{k\to+\infty} x_i(k) \geq \underline{x}_i, \qquad \liminf_{k\to+\infty} u_i(k) \geq \underline{u}_i, \quad i = 1,2$$

for any positive solution $(x_1(k), x_2(k), u_1(k), u_2(k))$ of system (1.3).

From (H_5) we can choose a constant $\varepsilon_1 > 0$ and a positive integer $k_2 \geq k_1$ such that

$$\sum_{s=k}^{k+\omega_1-1} \bigl(r_1(s) - a_{12}(s)\bar{x}_2 - d_1(s)\varepsilon_1\bigr) \geq \varepsilon_1 \quad \text{for all } k \geq k_2. \tag{3.5}$$

Consider the following auxiliary equation:

$$v(k+1) = \bigl(1 - e_1(k)\bigr)v(k) + f_1(k)\alpha_1, \tag{3.6}$$

where α_1 is a positive parameter. It follows from Lemma 2.3 that for $\varepsilon_1 > 0$ and $\bar{u}_1 > 0$ given above there exist positive constants $\hat{\delta}_1 = \hat{\delta}_1(\varepsilon_1)$ and $\hat{k}_0 = \hat{k}_0(\varepsilon_1, \bar{u}_1)$ such that, for any $k_0 \in Z$ and $0 \leq v_0 \leq \bar{u}_1$, when $f_1(k)\alpha_1 < \hat{\delta}_1$ for all $k \geq k_0$, we get

$$v(k, k_0, v_0) < \varepsilon_1 \quad \text{for all } k \geq k_0 + \hat{k}_0, \tag{3.7}$$

where $v(k, k_0, v_0)$ is the solution of equation (3.6) with the initial condition $v(k, k_0, v_0) = v_0$. By (3.5), we can find that there exists a positive constant $\alpha_1 \leq \min\{\varepsilon_1, \hat{\delta}_1/f_1^u\}$ such that

$$\sum_{s=k}^{k+\omega_1-1} \bigl(r_1(s) - a_{11}(s)\alpha_1 - a_{12}(s)\bar{x}_2 - b_1(s)\alpha_1\bar{x}_2 - d_1(s)\varepsilon_1\bigr) \geq \alpha_1 \quad \text{for all } k \geq k_2. \tag{3.8}$$

We first prove

$$\limsup_{k\to+\infty} x_1(k) \geq \alpha_1. \tag{3.9}$$

In fact, if this is not true, then there exists a positive solution $(x_1(k), x_2(k), u_1(k), u_2(k))$ of system (1.3) and a positive integer $k_3 > 0$ such that $x_1(k) < \alpha_1$ for all $k \geq k_3$. Further, from (3.2) and (3.4), we can find that there exists a positive integer $k_4 \geq k_3$ such that

$$x_i(k) \leq \bar{x}_i, \qquad u_1(k) \leq \bar{u}_1 \quad \text{for all } k \geq k_4, i = 1,2. \tag{3.10}$$

Thus, the third equation of system (1.3) implies

$$u_1(k+1) \leq \bigl(1 - e_1(k)\bigr)u_1(k) + f_1(k)\alpha_1 \quad \text{for all } k \geq k_3. \tag{3.11}$$

Let $v(k)$ be the solution of equation (3.6) with the initial value $v(k_4) = u_1(k_4)$. It follows from the comparison theorem for the difference equation and inequality (3.11) that

$$v(k) \leq u_1(k) \quad \text{for all } k \geq k_4. \tag{3.12}$$

In (3.7), we choose $k_0 = k_4$ and $v_0 = u_1(k_4)$. Since $f_1(k)\alpha_1 < \hat{\delta}_1$ for all $k \geq k_4$, we have

$$v(k) = v\bigl(k, k_4, u_1(k_4)\bigr) < \varepsilon_1 \quad \text{for all } k \geq k_4 + \hat{k}_0.$$

Further, by (3.12) we have

$$u_1(k) < \varepsilon_1 \quad \text{for all } k \geq k_4 + \hat{k}_0.$$

Therefore, $k \geq k_2 + k_4 + \hat{k}_0$ system (1.3) and (3.8) imply

$$\begin{aligned} x_1(k+\omega_1) &\geq x_1(k)\exp\left\{\sum_{s=k}^{k+\omega_1-1}\bigl[r_1(s) - a_{11}(s)\alpha_1 - a_{12}(s)\bar{x}_2 - b_1(s)\alpha_1\bar{x}_2 - d_1(s)\varepsilon_1\bigr]\right\} \\ &\geq x_1(k)\exp\{\alpha_1\}. \end{aligned}$$

Consequently, we further obtain

$$x_1(\bar{k} + n\omega_1) \geq x_1(\bar{k})\exp\{n\alpha_1\} \quad \text{for all } n \in Z,$$

where $\bar{k} = k_2 + k_4 + \hat{k}_0$, which implies $x_1(\bar{k} + n\omega_1) \to +\infty$ as $n \to +\infty$, which leads to a contradiction with (3.10). So (3.9) holds.

Next, we prove that there exists a positive constant $\underline{x}_1$ such that

$$\liminf_{k\to+\infty} x_1(k) \geq \underline{x}_1$$

for any positive solution $(x_1(k), x_2(k), u_1(k), u_2(k))$ of system (1.3). Otherwise, there exists a sequence with initial values $z^{(n)} = (\varphi_1^{(n)}, \varphi_2^{(n)}, \psi_1^{(n)}, \psi_2^{(n)})$ of system (1.3) such that

$$\liminf_{k\to+\infty} x_1\bigl(k, z^{(n)}\bigr) < \frac{\alpha_1}{n} \quad \text{for all } n = 1, 2, \ldots, \tag{3.13}$$

where $(x_1(k, z^{(n)}), x_2(k, z^{(n)}), u_1(k, z^{(n)}), u_2(k, z^{(n)}))$ is the solution of system (1.3) and satisfy $x_i(k) = \varphi_i^{(n)}(k)$, $u_i(k) = \psi_i^{(n)}(k)$, $i = 1, 2$.

It follows from (3.9) and (3.13) that there exist two sequences of positive integers $\{s_q^{(n)}\}$ and $\{t_q^{(n)}\}$ such that for each $n \in Z$

$$0 < s_1^{(n)} < t_1^{(n)} < s_2^{(n)} < t_2^{(n)} < \cdots < s_q^{(n)} < t_q^{(n)} < \cdots \tag{3.14}$$

and

$$s_q^{(n)} \to +\infty \quad \text{as } q \to +\infty \tag{3.15}$$

such that

$$x_1\bigl(s_q^{(n)}, z^{(n)}\bigr) > \alpha_1, \qquad x_1\bigl(t_q^{(n)}, z^{(n)}\bigr) < \frac{\alpha_1}{n} \tag{3.16}$$

and

$$\frac{\alpha_1}{n} \leq x_1\bigl(k, z^{(n)}\bigr) \leq \alpha_1 \quad \text{for all } k \in \bigl(s_q^{(n)}, t_q^{(n)}\bigr). \tag{3.17}$$

Equation (3.14) implies $t_q^{(n)} - s_q^{(n)} \geq 1$ for all $n \geq 1$. It follows from (3.2) and (3.4) that for each $n \in Z$ there exists an integer $k_4^{(n)} > k_4$ such that

$$x_i\big(k, z^{(n)}\big) \leq \bar{x}_i, u_1\big(k, z^{(n)}\big) \leq \bar{u}_1 \quad \text{for all } k \geq k_4^{(n)}, i = 1, 2.$$

From (3.15) we can choose an integer $k_1^{(n)}$ such that $s_q^{(n)} > k_4^{(n)}$ for all $q \geq k_1^{(n)}$. For any $k \in [s_q^{(n)}, t_q^{(n)} - 1]$ and $q \geq k_1^{(n)}$, we get

$$\begin{aligned} x_1\big(k+1, z^{(n)}\big) &= x_1\big(k, z^{(n)}\big) \exp\big\{r_1(k) - a_{11}(k)x_1\big(k, z^{(n)}\big) - a_{12}(k)x_2\big(k, z^{(n)}\big) \\ &\qquad - b_1(k)x_1\big(k, z^{(n)}\big)x_2\big(k, z^{(n)}\big) - d_1(k)u_1\big(k, z^{(n)}\big)\big\} \\ &\geq x_1\big(k, z^{(n)}\big) \exp\{-\theta\}, \end{aligned}$$

where $\theta = |r_1^l| + a_{11}^u\bar{x}_1 + a_{12}^u\bar{x}_2 + b_1^u\bar{x}_1\bar{x}_2 + d_1^u\bar{u}_1$. Further, by (3.16)

$$\begin{aligned} \frac{\alpha_1}{n} &> x_1\big(t_q^{(n)}, z^{(n)}\big) \\ &\geq x_1\big(s_q^{(n)}, z^{(n)}\big) \exp\big\{-\theta\big(t_q^{(n)} - s_q^{(n)}\big)\big\} \\ &> \alpha_1 \exp\big\{-\theta\big(t_q^{(n)} - s_q^{(n)}\big)\big\}, \end{aligned}$$

which implies

$$t_q^{(n)} - s_q^{(n)} > \frac{\ln n}{\theta} \quad \text{for all } q \geq k_1^{(n)}, n \in Z.$$

Obviously, $t_q^{(n)} - s_q^{(n)} \to \infty$ as $n \to \infty$. Hence, there exists an integer $N_0 > 0$ such that

$$t_q^{(n)} - s_q^{(n)} \geq \hat{k}_0 + k_2 + \omega_1 + 1 \quad \text{for all } n \geq N_0, q \geq k_1^{(n)}.$$

For all $k \in (s_q^{(n)}, t_q^{(n)})$, by (3.17) and the third equation of system (1.3) we get

$$u_1\big(k+1, z^{(n)}\big) \leq \big(1 - e_1(k)\big)u_1\big(k, z^{(n)}\big) + f_1(k)\alpha_1. \tag{3.18}$$

Let $v(n)$ be the solution of equation (3.6) with the initial value $v(s_q^{(n)} + 1) = u_1(s_q^{(n)} + 1)$. By applying the comparison theorem and inequality (3.18), we have

$$u_1\big(k, z^{(n)}\big) \leq v(k) \quad \text{for all } k \in \big(s_q^{(n)}, t_q^{(n)}\big). \tag{3.19}$$

In (3.7) we set $k_0 = s_q^{(n)} + 1$ and $v_0 = u_1(s_q^{(n)} + 1)$. Since $f_1(k)\alpha_1 < \hat{\delta}_1$ for all $k \in (s_q^{(n)}, t_q^{(n)})$, we have

$$v(k) = v\big(k, s_q^{(n)} + 1, u_1\big(s_q^{(n)} + 1\big)\big) < \varepsilon_1 \quad \text{for all } k \in \big[s_q^{(n)} + \hat{k}_0 + 1, t_q^{(n)}\big].$$

Therefore, (3.19) yields

$$u_1\big(k, z^{(n)}\big) < \varepsilon_1 \quad \text{for all } k \in \big[s_q^{(n)} + \hat{k}_0 + 1, t_q^{(n)}\big], n \geq N_0, q \geq k_1^{(n)}.$$

Hence, it follows from the first equation of system (1.3) that

$$x_1\bigl(k+1,z^{(n)}\bigr) > x_1\bigl(k,z^{(n)}\bigr)\exp\bigl\{r_1(s)-a_{11}(s)\alpha_1-a_{12}(s)\bar{x}_2-b_1(s)\alpha_1\bar{x}_2-d_1(s)\varepsilon_1\bigr\}.$$

Further, we have

$$x_1\bigl(k+\omega_1,z^{(n)}\bigr) > x_1\bigl(k,z^{(n)}\bigr)\exp\left\{\sum_{s=k}^{k+\omega_1-1}\bigl[r_1(s)-a_{11}(s)\alpha_1-a_{12}(s)\bar{x}_2-b_1(s)\alpha_1\bar{x}_2-d_1(s)\varepsilon_1\bigr]\right\}.$$

For any $n \geq N_0$, $q \geq k_1^{(n)}$ and $k \in [s_q^{(n)}+\hat{k}_0+1, t_q^{(n)}]$, (3.8), (3.16) and (3.17) yield

$$\begin{aligned}\frac{\alpha_1}{n} &> x_1\bigl(t_q^{(n)},z^{(n)}\bigr)\\ &> x_1\bigl(t_q^{(n)}-\omega_1,z^{(n)}\bigr)\exp\left\{\sum_{s=k}^{k+\omega_1-1}\bigl[r_1(s)-a_{11}(s)\alpha_1-a_{12}(s)\bar{x}_2-b_1(s)\alpha_1\bar{x}_2-d_1(s)\varepsilon_1\bigr]\right\}\\ &\geq \frac{\alpha_1}{n}\exp\{\alpha_1\},\end{aligned}$$

which leads to a contradiction. Therefore, there exists a positive constant $\underline{x}_1$ such that

$$\liminf_{k\to+\infty} x_1(k) \geq \underline{x}_1 \tag{3.20}$$

for any positive solution $(x_1(k), x_2(k), u_1(k), u_2(k))$ of system (1.3).

Similarly, we can also find that there exists a positive constant $\underline{x}_2$ such that

$$\liminf_{k\to+\infty} x_2(k) \geq \underline{x}_2 \tag{3.21}$$

for any positive solution $(x_1(k), x_2(k), u_1(k), u_2(k))$ of system (1.3).

From (3.20) and (3.21), we find, for any $\varepsilon > 0$ sufficiently small, that there exists a positive integer $\bar{k}_4$ such that

$$x_i(k) \leq \underline{x}_i - \varepsilon \quad \text{for all } q \geq \bar{k}_4. \tag{3.22}$$

It follows from (3.22) and the last two equations of system (1.3) that for all $q \geq \bar{k}_4$

$$u_i(k+1) \geq \bigl(1-e_i^u\bigr)u_i(k) + f_i^l(\underline{x}_i - \varepsilon), \quad i = 1,2. \tag{3.23}$$

By (H_1), (H_2) and Lemma 2.4, we have

$$\liminf_{k\to+\infty} u_i(k) \geq \frac{f_i^l(\underline{x}_i-\varepsilon)}{e_i^u}, \quad i = 1,2. \tag{3.24}$$

Letting $\varepsilon \to 0$, it follows from (3.24) that

$$\liminf_{k\to+\infty} u_i(k) \geq \frac{f_i^l \underline{x}_i}{e_i^u} \stackrel{\text{def}}{=} \underline{u}_i, \quad i = 1,2. \tag{3.25}$$

The proof of Theorem 3.2 is completed. □

Remark 3.1 Comparing with assumptions given by Chen and Chen [1], we can see our assumptions in Theorem 3.1 are more reasonable, and our result indicate that feedback control variables and toxic substances have no influence on the permanence of system (1.3).

Corollary 3.1 *If, in system* (1.3), $d_i(k) = e_i(k) = f_i(k) = 0$ $(i = 1, 2)$ *for* $k \in Z$, *then system* (1.3) *will be reduced to* (1.2). *Suppose that assumptions* (H_1), (H_3) *and* (H_5) *hold, then the system* (1.2) *has permanence*.

Remark 3.2 From Corollary 3.1, we can see that we improve the sufficient conditions which ensure the permanence of system (1.2) by Li and Chen [6] and Huo and Li [7]. We can also find that the toxic substances have no influence on the permanence of system (1.2).

4 Global stability

On the basis of permanence, further, we consider the stability of system (1.3) and obtain sufficient conditions for the global stability of system (1.3).

Theorem 4.1 *In addition to the conditions of Theorem* 3.2, *suppose*

$$
\begin{aligned}
(H_6)\quad & \lambda_i = \max\bigl\{\bigl|1-\bigl(a_{ii}^l + b_i^l \underline{x}_{3-i}\bigr)\underline{x}_i\bigr|, \bigl|1-\bigl(a_{ii}^u + b_i^u \bar{x}_{3-i}\bigr)\bar{x}_i\bigr|\bigr\} \\
& \quad + \bigl(a_{i3-i}^u + b_i^u \bar{x}_i\bigr)\bar{x}_{3-i} + d_i^u < 1, \quad i = 1, 2, \\
(H_7)\quad & \mu_i = 1 - e_i^l + f_i^u \bar{x}_i < 1, \quad i = 1, 2,
\end{aligned}
$$

then the system (1.3) *is globally stable*.

Proof Let $(x_1(k), x_2(k), u_1(k), u_2(k))$ and $(x_1^*(k), x_2^*(k), u_1^*(k), u_2^*(k))$ be any two positive solutions of system (1.3). Set

$$
y_i(k) = \ln x_i(k) - \ln x_i^*(k), \qquad v_i(k) = u_i(k) - u_i^*(k), \quad i = 1, 2.
$$

Next, we can only prove the following equations:

$$
\lim_{k\to+\infty} y_i(k) = 0, \qquad \lim_{k\to+\infty} v_i(k) = 0, \quad i = 1, 2.
$$

Since

$$
\begin{aligned}
y_i(k+1) &= \ln x_i(k+1) - \ln x_i^*(k+1) \\
&= \ln x_i(k) - \ln x_i^*(k) - a_{ii}(k)\bigl(x_i(k) - x_i^*(k)\bigr) - a_{i3-i}(k)\bigl(x_{3-i}(k) \\
&\quad - x_{3-i}^*(k)\bigr) - b_i(k)\bigl(x_i(k)x_{3-i}(k) - x_i^*(k)x_{3-i}^*(k)\bigr) - d_i(k)\bigl(u_i(k) - u_i^*(k)\bigr) \\
&= \bigl[1 - \bigl(a_{ii}(k) + b_i(k)x_{3-i}^*(k)\bigr)\theta_i(k)\bigr]\bigl(\ln x_i(k) - \ln x_i^*(k)\bigr) \\
&\quad - \bigl(a_{i3-i}(k) + b_i(k)x_i(k)\bigr)\theta_{3-i}(k)\bigl(\ln x_{3-i}(k) - \ln x_{3-i}^*(k)\bigr) \\
&\quad - d_i(k)\bigl(u_i(k) - u_i^*(k)\bigr) \\
&= \bigl[1 - \bigl(a_{ii}(k) + b_i(k)x_{3-i}^*(k)\bigr)\theta_i(k)\bigr]y_i(k) \\
&\quad - \bigl(a_{i3-i}(k) + b_i(k)x_i(k)\bigr)\theta_{3-i}(k)y_{3-i}(k) - d_i(k)v_i(k), \quad i = 1, 2.
\end{aligned}
\tag{4.1}
$$

Similarly,

$$v_i(k+1) = \big(1 - e_i(k)\big)v_i(k) + f_i(k)\theta_i(k)y_i(k), \quad i = 1,2, \tag{4.2}$$

where $\theta_i(k)$ lies between $x_i(k)$ and $x_i^*(k)$, $i = 1,2$.

It follows from (H_6) and (H_7) that there exists an $\varepsilon > 0$ such that

$$\begin{aligned} \lambda_i^* = \max\big\{&\big|1 - \big(a_{ii}^l + b_i^l(\underline{x}_{3-i} - \varepsilon)\big)(\underline{x}_i - \varepsilon)\big|, \\ &\big|1 - \big(a_{ii}^u + b_i^u(\bar{x}_{3-i} + \varepsilon)\big)(\bar{x}_i + \varepsilon)\big|\big\} \\ &+ \big(a_{i3-i}^u + b_i^u(\bar{x}_i + \varepsilon)\big)(\bar{x}_{3-i} + \varepsilon) + d_i^u < 1, \quad i = 1,2, \end{aligned} \tag{4.3}$$

$$\mu_i^* = 1 - e_i^l + f_i^u(\bar{x}_i + \varepsilon) < 1, \quad i = 1,2. \tag{4.4}$$

By Theorem 3.2, there exists a $k_5 \in Z$ such that

$$\underline{x}_i - \varepsilon \le x_i(k), \qquad x_i^*(k) \le \bar{x}_i + \varepsilon \quad \text{for all } k \ge k_5, i = 1,2.$$

Then we have

$$\underline{x}_i - \varepsilon \le \theta_i(k) \le \bar{x}_i + \varepsilon \quad \text{for all } k \ge k_5, i = 1,2.$$

From (4.1) and (4.2), we get

$$\begin{aligned} \big|y_i(k+1)\big| \le \max\big\{&\big|1 - \big(a_{ii}^l + b_i^l(\underline{x}_{3-i} - \varepsilon)\big)(\underline{x}_i - \varepsilon)\big|, \\ &\big|1 - \big(a_{ii}^u + b_i^u(\bar{x}_{3-i} + \varepsilon)\big)(\bar{x}_i + \varepsilon)\big|\big\}\big|y_i(k)\big| \\ &+ \big(a_{i3-i}^u + b_i^u(\bar{x}_i + \varepsilon)\big)(\bar{x}_{3-i} + \varepsilon)\big|y_{3-i}(k)\big| + d_i^u\big|v_i(k)\big|, \quad i = 1,2, \end{aligned} \tag{4.5}$$

$$\big|v_i(k+1)\big| \le \big(1 - e_i^l\big)\big|v_i(k)\big| + f_i^u(\bar{x}_i + \varepsilon)\big|y_i(k)\big|, \quad i = 1,2 \tag{4.6}$$

for all $k \ge k_5$.

Set $\lambda = \max\{\lambda_1^*, \lambda_2^*, \mu_1^*, \mu_2^*\}$, (4.3) and (4.4) imply $0 < \lambda < 1$.

It follows from (4.5) and (4.6) that

$$\begin{aligned} &\max\big\{\big|y_1(k+1)\big|, \big|y_2(k+1)\big|, \big|v_1(k+1)\big|, \big|v_2(k+1)\big|\big\} \\ &\quad\le \lambda \max\big\{\big|y_1(k)\big|, \big|y_2(k)\big|, \big|v_1(k)\big|, \big|v_2(k)\big|\big\} \end{aligned}$$

for all $k \ge k_5$. This yields

$$\begin{aligned} &\max\big\{\big|y_1(k)\big|, \big|y_2(k)\big|, \big|v_1(k)\big|, \big|v_2(k)\big|\big\} \\ &\quad\le \lambda^{k-k_5} \max\big\{\big|y_1(k_5)\big|, \big|y_2(k_5)\big|, \big|v_1(k_5)\big|, \big|v_2(k_5)\big|\big\}. \end{aligned}$$

Therefore

$$\lim_{k\to+\infty} y_i(k) = 0, \qquad \lim_{k\to+\infty} v_i(k) = 0, \quad i = 1,2.$$

The proof of Theorem 4.1 is completed. □

5 Extinction

In this section, we investigate the extinction property of the species in the system (1.3).

Theorem 5.1 *Suppose that assumptions* (H_1), (H_{31}) *and* (H_{41}) *hold, then we have*

$$\lim_{k\to+\infty} x_1(k) = 0$$

for any positive solution $(x_1(k), x_2(k), u_1(k), u_2(k))$ *of system* (1.3), *where* $(H_{31}) = \{(H_3)|i = 1\}$, $(H_{41}) = \{(H_4)|i = 1\}$.

Proof It follows from (H_{31}) that there exist a positive constant β and a positive integer S_0 such that

$$\sum_{s=k}^{k+\lambda_1-1} a_{11}(s) > \beta \quad \text{for all } k \geq S_0. \tag{5.1}$$

For any integer $k \geq S_0$ and $p > 0$, we can find that there exists an integer $q_p \geq 0$ such that

$$k + p\omega_1 - 1 \in \bigl(k + q_p\lambda_1 - 1, k + (q_p + 1)\lambda_1 - 1\bigr).$$

Therefore, (5.2) implies

$$\begin{aligned}\sum_{s=k}^{k+p\omega_1-1} a_{11}(s) &= \sum_{s=k}^{k+q_p\lambda_1-1} a_{11}(s) + \sum_{s=k+q_p\lambda_1}^{k+p\omega_1-1} a_{11}(s)\\ &> q_p\beta - \lambda_1 a_{11}^u.\end{aligned} \tag{5.2}$$

Since $q_p \to \infty$ as $p \to \infty$, there exist positive integers p_0 and $\lambda_1 > 0$ such that

$$q_{p_0}\beta - \lambda_1 a_{11}^u \geq \beta.$$

Thus, (5.2) yields

$$\sum_{s=k}^{k+p_0\omega_1-1} a_{11}(s) > \beta \quad \text{for all } k \geq S_0.$$

Hence, we can find that there exist integers $p_0 > 0$ and $\lambda_1 > 0$ such that

$$\liminf_{k\to\infty} \sum_{s=k}^{k+p_0\omega_1-1} a_{11}(s) > 0. \tag{5.3}$$

Similarly, it follows from (H_{41}) that

$$\limsup_{k\to\infty} \sum_{s=k}^{k+p_0\omega_1-1} r_1(s) \leq 0. \tag{5.4}$$

From (5.1) and (5.4), it follows that, for any $\varepsilon > 0$ sufficiently small, there exist a constant η and an integer $S_1 \geq S_0$ such that

$$\sum_{s=k}^{k+p_0\omega_1-1} \bigl[r_1(s) - a_{11}(s)\varepsilon\bigr] \leq -\eta \quad \text{for all } k \geq S_1. \tag{5.5}$$

Let $(x_1(k), x_2(k), u_1(k), u_2(k))$ be any positive solution of system (1.3). If, for all $\varepsilon > 0$, we have $x_1(k) \geq \varepsilon$ for all $k \geq S_1$.

Let $k_0 = S_1$, then (5.5) and the first equation of system (1.3) imply

$$\begin{aligned} x_1(k_0 + p_0\omega_1) &\leq x_1(k_0)\exp\left\{\sum_{s=k_0}^{k_0+p_0\omega_1-1} \bigl[r_1(s) - a_{11}(s)x_1(s)\bigr]\right\} \\ &\leq x_1(k_0)\exp\left\{\sum_{s=k_0}^{k_0+p_0\omega_1-1} \bigl[r_1(s) - a_{11}(s)\varepsilon\bigr]\right\} \\ &\leq x_1(k_0)\exp\{-\eta\}. \end{aligned}$$

Further, we have

$$x_1(k_0 + np_0\omega_1) \leq x_1(k_0)\exp\{-n\eta\} \quad \text{for all } n \in Z,$$

which implies $x_1(k_0 + np_0\omega_1) \to 0$ as $n \to \infty$. This leads to a contradiction. Hence, there exists an integer $k_1 \geq k_0$ such that $x_1(k_1) < \varepsilon$.

Next, we prove that

$$x_1(k) \leq \varepsilon \exp\bigl\{p_0\omega_1 r_1^u\bigr\} \quad \text{for all } k \geq k_1. \tag{5.6}$$

Otherwise, there exists an integer $k_2 \geq k_1$ such that $x_1(k) \leq \varepsilon \exp\{p_0\omega_1 r_1^u\}$ for all $k_1 \leq k \leq k_2$ and

$$x_1(k_2 + 1) > \varepsilon \exp\bigl\{p_0\omega_1 r_1^u\bigr\}. \tag{5.7}$$

We present two cases to prove (5.6).

Case 1. If $k_2 - k_1 < p_0\omega_1$, then from the first equation of system (1.3), we can obtain

$$\begin{aligned} x_1(k_2 + 1) &\leq x_1(k_1)\exp\left\{\sum_{s=k_1}^{k_2}\bigl[r_1(s) - a_{11}(s)x_1(s)\bigr]\right\} \\ &\leq x_1(k_1)\exp\left\{\sum_{s=k_1}^{k_2} r_1(s)\right\} \\ &\leq x_1(k_1)\exp\bigl\{(k_2 - k_1 + 1)r_1^u\bigr\} \\ &\leq \varepsilon \exp\bigl\{p_0\omega_1 r_1^u\bigr\}, \end{aligned}$$

which contradicts (5.7).

Case 2. If $k_2 - k_1 \geq p_0\omega_1$, let $k_2 = k_1 + np_0\omega_1 + \sigma$, where $n \in Z$ and $0 \leq \sigma < p_0\omega_1$, then (5.4) and the first equation of system (1.3) imply

$$\begin{aligned}
x_1(k_2+1) &\leq x_1(k_1)\exp\left\{\sum_{s=k_1}^{k_2}\bigl[r_1(s) - a_{11}(s)x_1(s)\bigr]\right\} \\
&\leq x_1(k_1)\exp\left\{\sum_{s=k_1}^{k_1+np_0\omega_1-1} r_1(s) + \sum_{s=k_1+np_0\omega_1}^{k_2} r_1(s)\right\} \\
&\leq x_1(k_1)\exp\left\{\sum_{s=k_1+np_0\omega_1}^{k_2} r_1(s)\right\} \\
&\leq \varepsilon\exp\bigl\{p_0\omega_1 r_1^u\bigr\},
\end{aligned}$$

which also leads to a contradiction with (5.7). According to the arguments of the two cases above, we find that (5.6) is true.

Letting $\varepsilon \to 0$, then (5.6) yields

$$\lim_{k\to+\infty} x_1(k) = 0.$$

Therefore, species x_1 in the system (1.3) is extinct. The proof of Theorem 5.1 is completed. □

Theorem 5.2 *Suppose that assumptions* (H_1), (H_{32}) *and* (H_{42}) *hold, then we have*

$$\lim_{k\to+\infty} x_2(k) = 0$$

for any positive solution $(x_1(k), x_2(k), u_1(k), u_2(k))$ *of system* (1.3), *where* $(\mathrm{H}_{32}) = \{(\mathrm{H}_3)|i = 2\}$, $(\mathrm{H}_{42}) = \{(\mathrm{H}_4)|i = 2\}$.

Proof The proof of Theorem 5.2 is similar to Theorem 5.1. So, here it is omitted. □

Corollary 5.1 *From Theorem* 5.1 *and Theorem* 5.2, *we can find that if assumptions* (H_1), (H_3) *and* (H_4) *hold, then*

$$\lim_{k\to+\infty} x_i(k) = 0, \quad i = 1, 2$$

for any positive solution $(x_1(k), x_2(k), u_1(k), u_2(k))$ *of system* (1.3).

If, in system (1.3), $d_i(k) = e_i(k) = f_i(k) = 0$ $(i = 1, 2)$ for $k \in Z$ then system (1.3) will be reduced to (1.2).

Corollary 5.2 *Suppose that assumptions in Theorem* 5.1 *hold, then*

$$\lim_{k\to+\infty} x_1(k) = 0$$

for any positive solution $(x_1(k), x_2(k))$ *of system* (1.2).

Suppose that assumptions in Theorem 5.2 *hold, then*

$$\lim_{k\to+\infty} x_2(k) = 0$$

for any positive solution $(x_1(k), x_2(k))$ *of system* (1.2).

Suppose that assumptions in Corollary 5.1 *hold, then*

$$\lim_{k\to+\infty} x_i(k) = 0, \quad i = 1, 2$$

for any positive solution $(x_1(k), x_2(k))$ *of system* (1.2).

Remark 5.1 Comparing with assumptions given in Chen and Chen [1], we can see that our assumptions in Theorem 5.2 are more reasonable. We can also find that feedback control variables and toxic substances have no influence on the extinction of system (1.3).

Remark 5.2 Comparing with assumptions given in Li and Chen [6], we can see that our assumptions in Corollary 5.2 are weaker. We can also find that toxic substances have no influence on the extinction of system (1.2).

6 Examples

The following examples show the feasibility of our main result.

Example 6.1 Consider the following system:

$$\begin{aligned}
x_1(k+1) &= x_1(k)\exp\left\{-1 + \frac{3}{k} - \bigl(1.8 - 0.2\cos(k)\bigr)x_1(k) - 0.8u_1(k)\right\} \\
&\quad - \bigl(0.7 - 0.1\sin(k)\bigr)x_2(k) - \bigl(1.5 + 0.4\cos(k)\bigr)x_1(k)x_2(k), \\
x_2(k+1) &= x_2(k)\exp\left\{0.9 - \frac{2}{k} - \bigl(0.8 - 0.1\sin(k)\bigr)x_1(k) - 0.4x_1(k)x_2(k)\right. \\
&\quad \left. - \bigl(1.2 - 0.4\cos(k)\bigr)x_2(k) - \bigl(1.1 + 0.5\cos(k)\bigr)u_2(k)\right\}, \\
u_1(k+1) &= 0.7u_1(k) + 0.2\bigl(1.5 + \sin(k)\bigr)x_1(k), \\
u_2(k+1) &= -0.2u_2(k) + 0.3\bigl(1.5 + \cos(k)\bigr)x_2(k).
\end{aligned} \tag{6.1}$$

Let $\omega = \lambda = 1$. By calculating, we obtain

$$\liminf_{k\to+\infty} \sum_{s=k}^{k+\lambda-1} a_{11}(s) \geq 1.6 > 0,$$

$$\limsup_{k\to+\infty} \sum_{s=k}^{k+\omega-1} r_1(s) = -1 < 0.$$

It is easy to see that the conditions in Theorem 5.1 holds. Therefore, x_1 in system (1.3) is extinct. Our numerical simulation supports this result (see Figure 1).

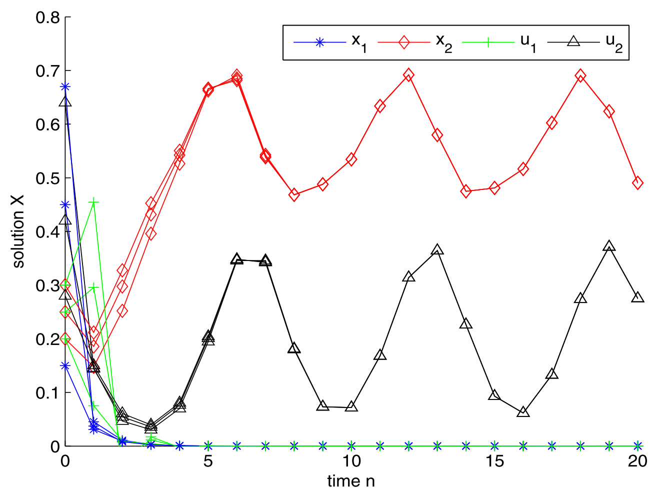

Figure 1 Dynamic behaviors of system (1.3) with initial condition $(x_1(0), x_2(0), u_1(0), u_2(0)) = (0.13, 0.35, 0.48, 0.62), (0.45, 0.75, 0.67, 0.42)$ and $(0.7, 0.55, 0.25, 0.75)$.

Example 6.2 Consider the following system:

$$\begin{aligned}
x_1(k+1) &= x_1(k)\exp\biggl\{1 - \frac{2}{k} - \bigl(1.8 - 0.2\cos(k)\bigr)x_1(k) - 0.8u_1(k) \\
&\qquad - \bigl(0.7 - 0.1\sin(k)\bigr)x_2(k) - \bigl(1.5 + 0.4\cos(k)\bigr)x_1(k)x_2(k)\biggr\}, \\
x_2(k+1) &= x_2(k)\exp\biggl\{-1 + \frac{3}{k} - \bigl(0.8 - 0.1\sin(k)\bigr)x_1(k) - 0.4x_1(k)x_2(k) \\
&\qquad - \bigl(1.2 - 0.4\cos(k)\bigr)x_2(k) - \bigl(1.1 + 0.5\cos(k)\bigr)u_2(k)\biggr\}, \\
u_1(k+1) &= -0.2u_1(k) + 0.2\bigl(3 + \sin(k)\bigr)x_1(k), \\
u_2(k+1) &= -0.1u_2(k) + 0.2\bigl(1.5 + \cos(k)\bigr)x_2(k).
\end{aligned} \tag{6.2}$$

Let $\omega = \lambda = 1$. By calculating, we obtain

$$\liminf_{k\to+\infty} \sum_{s=k}^{k+\lambda-1} a_{22}(s) \geq 0.8 > 0,$$

$$\limsup_{k\to+\infty} \sum_{s=k}^{k+\omega-1} r_2(s) = -1 < 0.$$

It is easy to see that the conditions in Theorem 5.2 hold. Therefore, x_2 in system (1.3) is extinct. Our numerical simulation supports this result (see Figure 2).

Example 6.3 Consider the following system:

$$\begin{aligned}
x_1(k+1) &= x_1(k)\exp\biggl\{-1 + \frac{2}{k} - \bigl(1.8 - 0.2\cos(k)\bigr)x_1(k) - 0.8u_1(k) \\
&\qquad - \bigl(0.7 - 0.1\sin(k)\bigr)x_2(k) - \bigl(1.5 + 0.4\cos(k)\bigr)x_1(k)x_2(k)\biggr\},
\end{aligned}$$

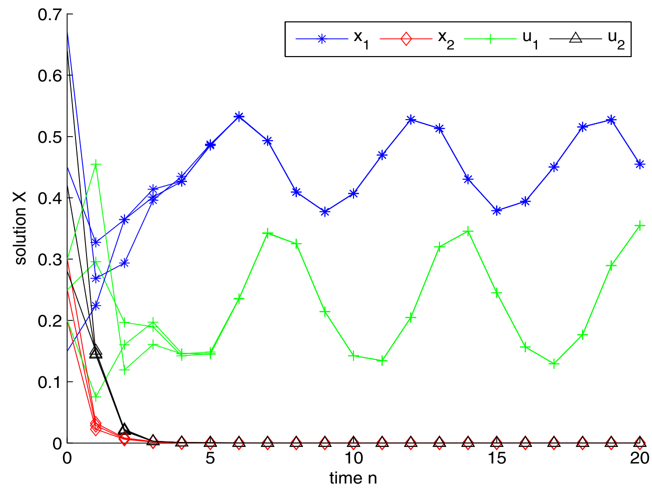

Figure 2 Dynamic behaviors of system (1.3) with initial condition $(x_1(0), x_2(0), u_1(0), u_2(0)) = (0.45, 0.25, 0.67, 0.42), (0.15, 0.2, 0.3, 0.64)$ and $(0.67, 0.3, 0.55, 0.28)$.

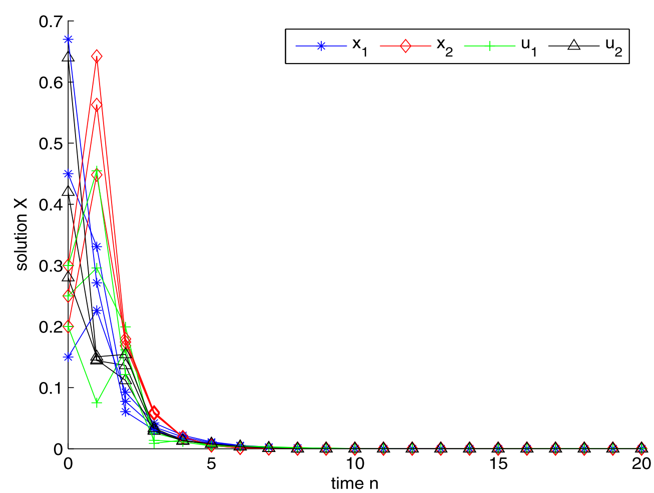

Figure 3 Dynamic behaviors of system (1.3) with initial condition $(x_1(0), x_2(0), u_1(0), u_2(0)) = (0.45, 0.25, 0.67, 0.42), (0.15, 0.2, 0.3, 0.64)$ and $(0.67, 0.3, 0.55, 0.28)$.

$$\begin{aligned} x_2(k+1) &= x_2(k)\exp\biggl\{-2+\frac{4}{k}-\bigl(0.8-0.1\sin(k)\bigr)x_1(k)-0.4x_1(k)x_2(k) \\ &\qquad -\bigl(1.2-0.4\cos(k)\bigr)x_2(k)-\bigl(1.1+0.5\cos(k)\bigr)u_2(k)\biggr\}, \\ u_1(k+1) &= -0.2u_1(k)+0.2\bigl(3+\sin(k)\bigr)x_1(k), \\ u_2(k+1) &= 0.1u_2(k)+0.2\bigl(1.5+\cos(k)\bigr)x_2(k). \end{aligned} \tag{6.3}$$

Let $\omega = \lambda = 1$. By calculating, we obtain

$$\liminf_{k\to+\infty}\sum_{s=k}^{k+\lambda-1} a_{11}(s) \ge 1.6 > 0, \qquad \limsup_{k\to+\infty}\sum_{s=k}^{k+\omega-1} r_1(s) = -1 < 0,$$

$$\liminf_{k\to+\infty}\sum_{s=k}^{k+\lambda-1} a_{22}(s) = 0.8 > 0, \qquad \limsup_{k\to+\infty}\sum_{s=k}^{k+\omega-1} r_1(s) = -2 < 0.$$

It is easy to see that the conditions in the corollary hold. Therefore, x_1 and x_2 in system (1.3) are extinct. Our numerical simulation supports this result (see Figure 3).

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

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