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Global attractivity of an integro-differential model of competition

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Abstract

An integro-differential model of competition is studied in this paper. We show that the system always admits a unique globally attractive positive equilibrium. Our result complements and supplements the main result in the literature.

MSC: competition model; integro-differential equation; delay; global attractivity

Keywords: 34C05; 92D25; 34D20; 34D40

1 Introduction

During the last decades, many scholars investigated the dynamic behaviors of the cooperative system, or mutualism model, see [1–36] and the references cited therein. Specially, in a series of their studies, Chen and his coauthors ([1–6, 28]) argued that a suitable cooperative model should consider the saturating effect of the relationship between the species, and they gave a thorough investigation on the dynamic behaviors of the cooperative system with Holling II type functional response.

Stimulated by the works of [1–6], in [28], Yang et al. studied the dynamic behaviors of the following autonomous discrete cooperative system:

$$
\begin{aligned}
x_1(k+1) &= x_1(k)\exp\left\{r_1\left[\frac{K_1+\alpha_1 x_2(k)}{1+x_2(k)}-x_1(k)\right]\right\},\\
x_2(k+1) &= x_2(k)\exp\left\{r_2\left[\frac{K_2+\alpha_2 x_1(k)}{1+x_1(k)}-x_2(k)\right]\right\},
\end{aligned}
$$

where $x_i(k)$ $(i=1,2)$ is the population density of the ith species at k-generation. They showed that if

(H_1) r_i, K_i, α_i $(i=1,2)$ are all positive constants and $\alpha_i > K_i$ $(i=1,2)$;

and

(H_2) $r_i\alpha_i \le 1$ $(i=1,2)$

hold, then the above system admits a unique positive equilibrium (x_1^*, x_2^*), which is globally asymptotically stable.

Recently, Chen [36] argued that it is interesting to investigate the dynamic behaviors of the above system under the assumption $K_i > \alpha_i$. He showed that if $r_iK_i \le 1$, $i=1,2$, the system could also admit a unique globally attractive positive equilibrium.

In [35], Chen further proposed the following discrete competition model:

$$x_1(k+1) = x_1(k)\exp\left\{r_1(k)\left[\frac{K_1(k) + \alpha_1(k)\sum_{s=0}^{+\infty} J_2(s)x_2(k-s)}{1 + \sum_{s=0}^{+\infty} J_2(s)x_2(k-s)} - x_1\big(k - \delta_1(k)\big)\right]\right\},$$
$$x_2(k+1) = x_2(k)\exp\left\{r_2(k)\left[\frac{K_2(k) + \alpha_2(k)\sum_{s=0}^{+\infty} J_1(s)x_1(k-s)}{1 + \sum_{s=0}^{+\infty} J_1(s)x_1(k-s)} - x_2\big(k - \delta_2(k)\big)\right]\right\},$$

where r_i, K_i, α_i, τ_i and δ_i, $i = 1,2$, are all nonnegative sequences bounded above and below by positive constants, and $K_i > \alpha_i$, $i = 1,2$. Sufficient conditions are obtained for the permanence of the above system.

The success of [35, 36] motivated us to consider the continuous case. Xie et al. [14] have already investigated the stability property of the following two species mutualism model:

$$\begin{aligned}\frac{dN_1(t)}{dt} &= r_1N_1(t)\left[\frac{K_1 + \alpha_1\int_0^\infty J_2(s)N_2(t-s)\,ds}{1 + \int_0^\infty J_2(s)N_2(t-s)\,ds} - N_1(t)\right],\\ \frac{dN_2(t)}{dt} &= r_2N_2(t)\left[\frac{K_2 + \alpha_2\int_0^\infty J_1(s)N_1(t-s)\,ds}{1 + \int_0^\infty J_1(s)N_1(t-s)\,ds} - N_2(t)\right],\end{aligned}\tag{1.1}$$

where r_i, K_i and α_i, $i = 1,2$, are all continuous positive constants, $J_i \in C([0,+\infty),[0,+\infty))$ and $\int_0^\infty J_i(s)\,ds = 1$, $i = 1,2$. Under the assumption $\alpha_i > K_i$, $i = 1,2$, by using the iterative method, Xie et al. [14] showed that the system admits a unique globally attractive positive equilibrium. For more background of system (1.1), one could refer to [1, 14, 20] and the references cited therein.

As far as system (1.1) is concerned, one interesting issue is proposed: What would happen if $\alpha_i < K_i$, $i = 1,2$? Is it possible that the system admits dynamic behaviors similar to those of the case $\alpha_i > K_i$, $i = 1,2$?

Note that under the assumption $\alpha_i < K_i$, $i = 1,2$, the first equation in system (1.1) can be rewritten as follows:

$$\begin{aligned}\frac{dN_1(t)}{dt} &= r_1N_1(t)\left[\frac{K_1 + \alpha_1\int_0^\infty J_2(s)N_2(t-s)\,ds}{1 + \int_0^\infty J_2(s)N_2(t-s)\,ds} - N_1(t)\right]\\ &= r_1N_1(t)\left[\frac{K_1(1 + \int_0^\infty J_2(s)N_2(t-s)\,ds)}{1 + \int_0^\infty J_2(s)N_2(t-s)\,ds} - N_1(t)\right.\\ &\quad\left. - \frac{(K_1 - \alpha_1)\int_0^\infty J_2(s)N_2(t-s)\,ds)}{1 + \int_0^\infty J_2(s)N_2(t-s)\,ds}\right]\\ &= r_1N_1(t)\left[K_1 - N_1(t) - \frac{(K_1 - \alpha_1)\int_0^\infty J_2(s)N_2(t-s)\,ds}{1 + \int_0^\infty J_2(s)N_2(t-s)\,ds}\right].\end{aligned}\tag{1.2}$$

Similarly, the second equation in system (1.1) can be rewritten as follows:

$$\frac{dN_2(t)}{dt} = r_2N_2(t)\left[K_2 - N_2(t) - \frac{(K_2 - \alpha_2)\int_0^\infty J_1(s)N_1(t-s)\,ds}{1 + \int_0^\infty J_1(s)N_1(t-s)\,ds}\right].\tag{1.3}$$

From (1.2) and (1.3) one could easily see that the first species has a negative effect on the second species, and the second species has a negative effect on the first species. Therefore, under the assumption $\alpha_i < K_i$, $i = 1,2$, the relationship between the two species is competition.

From the point of view of biology, in the sequel, we shall consider (1.2)-(1.3) together with the initial conditions

$$N_i(s) = \phi_i(s), \quad s \in (-\infty, 0], i = 1, 2, \tag{1.4}$$

where $\phi_i \in BC^+$ and

$$BC^+ = \{\phi \in C((-\infty, 0], [0, +\infty)) : \phi(0) > 0 \text{ and } \phi \text{ is bounded}\}, \quad i = 1, 2.$$

One could easily prove that $N_i(t) > 0$ for all $i = 1, 2$ in a maximal interval of the existence of solution.

The aim of this paper is to give an affirmative answer to the above issue. More precisely, we will prove the following result.

Theorem 1.1 *Under the assumption $\alpha_i < K_i$, $i = 1, 2$, system* (1.1) *with the initial conditions* (1.4) *admits a unique positive equilibrium (N_1^*, N_2^*), which is globally attractive, that is, for any positive solution $(N_1(t), N_2(t))$ of system* (1.1) *with the initial conditions* (1.4), *one has*

$$\lim_{t \to +\infty} N_i(t) = N_i^*, \quad i = 1, 2.$$

2 Proof of the main result

Now let us state several lemmas which will be useful in proving the main result.

Lemma 2.1 *System* (1.1) *admits a unique positive equilibrium (N_1^*, N_2^*).*

Proof The positive equilibrium of system (1.1) satisfies the following equation:

$$\begin{aligned} &\frac{K_1 + \alpha_1 N_2}{1 + N_2} - N_1 = 0, \\ &\frac{K_2 + \alpha_2 N_1}{1 + N_1} - N_2 = 0, \end{aligned} \tag{2.1}$$

which is equivalent to

$$\begin{aligned} &A_1 N_1^2 + A_2 N_1 + A_3 = 0, \\ &B_1 N_2^2 + B_2 N_2 + B_3 = 0, \end{aligned} \tag{2.2}$$

where

$$\begin{aligned} &A_1 = 1 + \alpha_2, \qquad A_2 = K_2 - K_1 - \alpha_2\alpha_1 + 1, \qquad A_3 = -\alpha_1 K_2 - K_1; \\ &B_1 = 1 + \alpha_1, \qquad B_2 = K_1 - K_2 - \alpha_1\alpha_2 + 1, \qquad B_3 = -K_2 - \alpha_2 K_1. \end{aligned}$$

Noting that $A_1 > 0$, $A_3 < 0$, $B_1 > 0$, $B_3 < 0$, it immediately follows that system (2.1) admits a unique positive solution (N_1^*, N_2^*), where

$$N_1^* = \frac{-A_2 + \sqrt{A_2^2 - 4A_1A_3}}{2A_1}, \qquad N_2^* = \frac{-B_2 + \sqrt{B_2 - 4B_1B_3}}{2B_1}.$$

This ends the proof of Lemma 2.1. □

Lemma 2.2 ([27]) *Let $x : R \to R$ be a bounded nonnegative continuous function, and let $k : [0, +\infty) \to (0, +\infty)$ be a continuous kernel such that $\int_0^\infty k(s)\,ds = 1$. Then*

$$\begin{aligned}\liminf_{t\to+\infty} x(t) &\le \liminf_{t\to+\infty} \int_{-\infty}^{t} k(t-s)x(s)\,ds \\ &\le \limsup_{t\to+\infty} \int_{-\infty}^{t} k(t-s)x(s)\,ds \le \limsup_{t\to+\infty} x(t).\end{aligned}$$

Lemma 2.3 ([7]) *If $a > 0$, $b > 0$ and $\dot{x} \ge x(b - ax)$, when $t \ge 0$ and $x(0) > 0$, we have*

$$\liminf_{t\to+\infty} x(t) \ge \frac{b}{a}.$$

If $a > 0$, $b > 0$ and $\dot{x} \le x(b - ax)$, when $t \ge 0$ and $x(0) > 0$, we have

$$\limsup_{t\to+\infty} x(t) \le \frac{b}{a}.$$

Now we are in a position to prove the main result of this paper.

Proof of Theorem 1.1 Let $(N_1(t), N_2(t))$ be any positive solution of system (1.1) with initial conditions (1.4). From (1.2) it follows that

$$\frac{dN_1(t)}{dt} \le r_1 N_1(t)\bigl(K_1 - N_1(t)\bigr). \tag{2.3}$$

Thus, as a direct corollary of Lemma 2.3, according to (2.3), one has

$$\limsup_{t\to+\infty} N_1(t) \le K_1, \tag{2.4}$$

and so, from Lemma 2.2 we have

$$\limsup_{t\to+\infty} \int_{-\infty}^{t} J_1(t-s)N_1(s)\,ds \le K_1. \tag{2.5}$$

Hence, for $\varepsilon > 0$ small enough, it follows from (2.4) and (2.5) that there exists $T_{11} > 0$ such that for all $t \ge T_{11}$,

$$\begin{aligned}&N_1(t) < K_1 + \varepsilon \stackrel{\text{def}}{=} M_1^{(1)}, \\ &\int_0^\infty J_1(s)N_1(t-s)\,ds = \int_{-\infty}^{t} J_1(t-s)N_1(s)\,ds < K_1 + \varepsilon \stackrel{\text{def}}{=} M_1^{(1)}.\end{aligned} \tag{2.6}$$

Similarly, for above $\varepsilon > 0$, it follows from (1.3) that

$$\begin{aligned}&\limsup_{t\to+\infty} N_2(t) \le K_2, \\ &\limsup_{t\to+\infty} \int_{-\infty}^{t} J_2(t-s)N_2(s)\,ds \le K_2,\end{aligned}$$

and so, there exists $T_{12} > T_{11}$ such that for all $t \geq T_{12}$,

$$\begin{aligned} &N_2(t) < K_2 + \varepsilon \overset{\text{def}}{=} M_2^{(1)}, \\ &\int_0^{\infty} J_2(s)N_2(t-s)\,ds = \int_{-\infty}^{t} J_2(t-s)N_2(s)\,ds < K_2 + \varepsilon \overset{\text{def}}{=} M_2^{(1)}. \end{aligned} \tag{2.7}$$

Also, from (1.2) we have

$$\begin{aligned} \frac{dN_1(t)}{dt} &= r_1N_1(t)\left[K_1 - N_1(t) - \frac{(K_1-\alpha_1)\int_0^{\infty} J_2(s)N_2(t-s)\,ds}{1+\int_0^{\infty} J_2(s)N_2(t-s)\,ds}\right] \\ &\geq r_1N_1(t)\left[K_1 - N_1(t) - \frac{(K_1-\alpha_1)(1+\int_0^{\infty} J_2(s)N_2(t-s)\,ds)}{1+\int_0^{\infty} J_2(s)N_2(t-s)\,ds}\right] \\ &= r_1N_1(t)\bigl[K_1 - N_1(t) - (K_1-\alpha_1)\bigr] \\ &= r_1N_1(t)\bigl[\alpha_1 - N_1(t)\bigr]. \end{aligned} \tag{2.8}$$

Thus, as a direct corollary of Lemma 2.3, according to (2.8), one has

$$\liminf_{t\to+\infty} N_1(t) \geq \alpha_1, \tag{2.9}$$

and so, from Lemma 2.2 we have

$$\liminf_{t\to+\infty} \int_{-\infty}^{t} J_1(t-s)N_1(s)\,ds \geq \alpha_1. \tag{2.10}$$

Hence, for $\varepsilon > 0$ small enough, without loss of generality, we may assume that $\varepsilon < \frac{1}{2}\min\{\alpha_1,\alpha_2\}$. It follows from (2.9) and (2.10) that there exists $T_{13} > T_{12}$ such that for all $t \geq T_{13}$,

$$\begin{aligned} &N_1(t) > \alpha_1 - \varepsilon \overset{\text{def}}{=} m_1^{(1)}, \\ &\int_0^{\infty} J_1(s)N_1(t-s)\,ds = \int_{-\infty}^{t} J_1(t-s)N_1(s)\,ds > \alpha_1 - \varepsilon \overset{\text{def}}{=} m_1^{(1)}. \end{aligned} \tag{2.11}$$

Similarly, for above $\varepsilon > 0$, it follows from (1.3) that

$$\begin{aligned} &\liminf_{t\to+\infty} N_2(t) \geq \alpha_2, \\ &\liminf_{t\to+\infty} \int_{-\infty}^{t} J_2(t-s)N_2(s)\,ds \geq \alpha_2, \end{aligned}$$

and so, there exists $T_{14} > T_{13}$ such that for all $t \geq T_{14}$,

$$\begin{aligned} &N_2(t) > \alpha_2 - \varepsilon \overset{\text{def}}{=} m_2^{(1)}, \\ &\int_0^{\infty} J_2(s)N_2(t-s)\,ds = \int_{-\infty}^{t} J_2(t-s)N_2(s)\,ds > \alpha_2 - \varepsilon \overset{\text{def}}{=} m_2^{(1)}. \end{aligned} \tag{2.12}$$

It follows from (2.6), (2.7), (2.11) and (2.12) that for all $t \geq T_{14}$,

$$0 < m_1^{(1)} < x(t) < M_1^{(1)}, \qquad 0 < m_2^{(1)} < y(t) < M_2^{(1)}. \tag{2.13}$$

Note that the function $g(x) = \frac{x}{1+x}$ ($x \geq 0$) is a strictly increasing function, hence (2.12) together with (1.2) implies

$$\frac{dN_1(t)}{dt} < r_1 N_1(t)\left[K_1 - N_1(t) - \frac{(K_1-\alpha_1)m_2^{(1)}}{1+m_2^{(1)}}\right] \quad \text{for } t > T_{14}. \tag{2.14}$$

Therefore, by Lemma 2.3, we have

$$\limsup_{t\to+\infty} N_1(t) \leq K_1 - \frac{(K_1-\alpha_1)m_2^{(1)}}{1+m_2^{(1)}} = \frac{K_1+\alpha_1 m_2^{(1)}}{1+m_2^{(1)}}. \tag{2.15}$$

From Lemma 2.2 and (2.15) we have

$$\limsup_{t\to+\infty} \int_{-\infty}^{t} J_1(t-s)N_1(s)\,ds \leq \frac{K_1+\alpha_1 m_2^{(1)}}{1+m_2^{(1)}}. \tag{2.16}$$

That is, there exists $T_{21} > T_{14}$ such that for all $t \geq T_{21}$,

$$\begin{aligned} &N_1(t) < \frac{K_1+\alpha_1 m_2^{(1)}}{1+m_2^{(1)}} + \frac{\varepsilon}{2} \stackrel{\text{def}}{=} M_1^{(2)}, \\ &\int_0^{\infty} J_1(s)N_1(t-s)\,ds = \int_{-\infty}^{t} J_1(t-s)N_1(s)\,ds < \frac{K_1+\alpha_1 m_2^{(1)}}{1+m_2^{(1)}} + \frac{\varepsilon}{2} \stackrel{\text{def}}{=} M_1^{(2)}. \end{aligned} \tag{2.17}$$

Similar to the analysis of (2.14)-(2.17), from (2.11) and (1.3), there exists $T_{22} > T_{21}$ such that for all $t \geq T_{22}$,

$$\begin{aligned} &N_2(t) < \frac{K_2+\alpha_2 m_1^{(1)}}{1+m_1^{(1)}} + \frac{\varepsilon}{2} \stackrel{\text{def}}{=} M_2^{(2)}, \\ &\int_0^{\infty} J_2(s)N_2(t-s)\,ds < \frac{K_2+\alpha_2 m_1^{(1)}}{1+m_1^{(1)}} + \frac{\varepsilon}{2} \stackrel{\text{def}}{=} M_2^{(2)}. \end{aligned} \tag{2.18}$$

It follows from (2.6), (2.7), (2.17) and (2.18) that

$$M_1^{(2)} < M_1^{(1)}, \qquad M_2^{(2)} < M_2^{(1)}. \tag{2.19}$$

Again from the strictly increasing function $g(x) = \frac{x}{1+x}$ ($x \geq 0$) and (1.2), it follows that

$$\frac{dN_1(t)}{dt} > r_1 N_1(t)\left[K_1 - N_1(t) - \frac{(K_1-\alpha_1)M_2^{(1)}}{1+M_2^{(1)}}\right] \quad \text{for } t > T_{22}. \tag{2.20}$$

Therefore, by Lemma 2.3, we have

$$\liminf_{t\to+\infty} N_1(t) \geq K_1 - \frac{(K_1-\alpha_1)M_2^{(1)}}{1+M_2^{(1)}} = \frac{K_1+\alpha_1 M_2^{(1)}}{1+M_2^{(1)}}. \tag{2.21}$$

Thus, from Lemma 2.2 we have

$$\liminf_{t\to+\infty} \int_{-\infty}^{t} J_1(t-s)N_1(s)\,ds \geq \frac{K_1+\alpha_1 M_2^{(1)}}{1+M_2^{(1)}}. \tag{2.22}$$

That is, there exists $T_{23} > T_{22}$ such that for all $t \geq T_{23}$,

$$\begin{aligned} &N_1(t) > \frac{K_1 + \alpha_1 M_2^{(1)}}{1 + M_2^{(1)}} - \frac{\varepsilon}{2} \stackrel{\text{def}}{=} m_1^{(2)}, \\ &\int_0^\infty J_1(s) N_1(t-s)\,ds = \int_{-\infty}^t J_1(t-s) N_1(s)\,ds > \frac{K_1 + \alpha_1 M_2^{(1)}}{1 + M_2^{(1)}} - \frac{\varepsilon}{2} \stackrel{\text{def}}{=} m_1^{(2)}. \end{aligned} \tag{2.23}$$

Similar to the analysis of (2.20)-(2.23), from (2.13) and (1.3), there exists $T_{24} > T_{23}$ such that for all $t \geq T_{24}$,

$$\begin{aligned} &N_2(t) > \frac{K_2 + \alpha_2 M_1^{(1)}}{1 + M_1^{(1)}} - \frac{\varepsilon}{2} \stackrel{\text{def}}{=} m_2^{(2)}, \\ &\int_0^\infty J_2(s) N_2(t-s)\,ds > \frac{K_2 + \alpha_2 M_1^{(1)}}{1 + M_1^{(1)}} - \frac{\varepsilon}{2} \stackrel{\text{def}}{=} m_2^{(2)}. \end{aligned} \tag{2.24}$$

Obviously,

$$K_1 - \alpha_1 > (K_1 - \alpha_1)\frac{M_2^{(1)}}{1 + M_2^{(1)}};$$

consequently,

$$K_1 - (K_1 - \alpha_1) < K_1 - (K_1 - \alpha_1)\frac{M_2^{(1)}}{1 + M_2^{(1)}},$$

and so, it follows from (2.11) and (2.23) that

$$m_1^{(2)} > m_1^{(1)}. \tag{2.25}$$

Similarly, it follows from (2.12) and (2.24) that

$$m_2^{(2)} > m_2^{(1)}. \tag{2.26}$$

Repeating the above procedure, we get four sequences $M_i^{(n)}$, $m_i^{(n)}$, $i = 1, 2$, $n = 1, 2, \ldots$, such that for $n \geq 2$

$$\begin{aligned} &M_1^{(n)} = \frac{K_1 + \alpha_1 m_2^{(n-1)}}{1 + m_2^{(n-1)}} + \frac{\varepsilon}{n}; \qquad M_2^{(n)} = \frac{K_2 + \alpha_2 m_1^{(n-1)}}{1 + m_1^{(n-1)}} + \frac{\varepsilon}{n}; \\ &m_1^{(n)} = \frac{K_1 + \alpha_1 M_2^{(n-1)}}{1 + M_2^{(n-1)}} - \frac{\varepsilon}{n}; \qquad m_2^{(n)} = \frac{K_2 + \alpha_2 M_1^{(n-1)}}{1 + M_1^{(n-1)}} - \frac{\varepsilon}{n}. \end{aligned} \tag{2.27}$$

Obviously,

$$m_i^{(n)} < N_i(t) < M_i^{(n)} \quad \text{for } t \geq T_{n4}, i = 1, 2.$$

We claim that sequences $M_i^{(n)}$, $i = 1, 2$, are nonincreasing, and sequences $m_i^{(n)}$, $i = 1, 2$, are nondecreasing. To prove this claim, we will carry on by induction. Firstly, from (2.25), (2.26) and (2.19), we have

$$M_i^{(2)} < M_i^{(1)}, \qquad m_i^{(2)} > m_i^{(1)}, \quad i = 1, 2.$$

Let us assume now that our claim is true for n, that is,

$$M_i^{(n)} < M_i^{(n-1)}, \qquad m_i^{(n)} > m_i^{(n-1)}, \quad i = 1, 2.$$

Let us consider the function $g_i(x) = \frac{K_i + \alpha_i x}{1+x}$ $(K_i > \alpha_i, i = 1, 2)$ since

$$g_i^{'}(x) = -\frac{K_i - \alpha_i}{(1+x)^2} < 0,$$

then $g_i(x)$, $i = 1, 2$, is a strictly decreasing function of x. From the monotonic property of $g_i(x)$, $i = 1, 2$, it immediately follows that

$$\begin{aligned}
M_1^{(n+1)} &= \frac{K_1 + \alpha_1 m_2^{(n)}}{1 + m_2^{(n)}} + \frac{\varepsilon}{n+1} < \frac{K_1 + \alpha_1 m_2^{(n-1)}}{1 + m_2^{(n-1)}} + \frac{\varepsilon}{n} = M_1^{(n)}; \\
M_2^{(n+1)} &= \frac{K_2 + \alpha_2 m_1^{(n)}}{1 + m_1^{(n)}} + \frac{\varepsilon}{n+1} < \frac{K_2 + \alpha_2 m_1^{(n-1)}}{1 + m_1^{(n-1)}} + \frac{\varepsilon}{n} = M_2^{(n)}; \\
m_1^{(n+1)} &= \frac{K_1 + \alpha_1 M_2^{(n)}}{1 + M_2^{(n)}} - \frac{\varepsilon}{n+1} > \frac{K_1 + \alpha_1 M_2^{(n-1)}}{1 + M_2^{(n-1)}} - \frac{\varepsilon}{n} = m_1^{(n)}; \\
m_2^{(n+1)} &= \frac{K_2 + \alpha_2 M_1^{(n)}}{1 + M_1^{(n)}} - \frac{\varepsilon}{n+1} > \frac{K_2 + \alpha_2 M_1^{(n-1)}}{1 + M_1^{(n-1)}} - \frac{\varepsilon}{n} = m_2^{(n)}.
\end{aligned}$$

Therefore,

$$\lim_{t\to+\infty} M_i^{(n)} = \overline{N}_i, \qquad \lim_{t\to+\infty} m_i^{(n)} = \underline{N}_i, \quad i = 1, 2.$$

Letting $n \to +\infty$ in (2.27), we obtain

$$\begin{aligned}
\overline{N}_1 &= \frac{K_1 + \alpha_1 \underline{N}_2}{1 + \underline{N}_2}; \qquad \overline{N}_2 = \frac{K_2 + \alpha_2 \underline{N}_1}{1 + \underline{N}_1}; \\
\underline{N}_1 &= \frac{K_1 + \alpha_1 \overline{N}_2}{1 + \overline{N}_2}; \qquad \underline{N}_2 = \frac{K_2 + \alpha_2 \overline{N}_1}{1 + \overline{N}_1};
\end{aligned} \tag{2.28}$$

(2.28) shows that $(\overline{N}_1, \underline{N}_2)$ and $(\underline{N}_1, \overline{N}_2)$ are solutions of (2.1). By Lemma 2.1, (2.1) has a unique positive solution $E^*(N_1^*, N_2^*)$. Hence, we conclude that

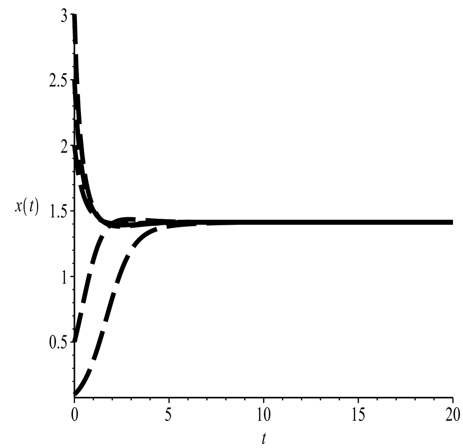
$$\overline{N}_i = \underline{N}_i = N_i^*, \quad i = 1, 2,$$

that is,

$$\lim_{t\to+\infty} N_i(t) = N_i^*, \quad i = 1, 2.$$

Thus, the unique interior equilibrium $E^*(N_1^*, N_2^*)$ is globally attractive. This completes the proof of Theorem 1.1. □

Figure 1 Dynamic behavior of the first species in system (3.1) with the initial conditions $(x(s), y(s)) = (0.5, 0.5), (0.1, 1), (2, 2), (2.5, 2.5)$ and $(3, 3)$, $s \in (-\infty, 0]$, respectively.



3 Numerical simulations

In this section we will give several examples to show the feasibility of Theorem 1.1. Firstly, let us consider the weakly integral kernel case.

Example 3.1

$$\begin{aligned} \frac{dx(t)}{dt} &= x(t)\left[\frac{2 + \int_0^\infty e^{-s} y(t-s)\,ds}{1 + \int_0^\infty e^{-s} y(t-s)\,ds} - x(t)\right], \\ \frac{dy(t)}{dt} &= y(t)\left[\frac{2 + \int_0^\infty e^{-s} x(t-s)\,ds}{1 + \int_0^\infty e^{-s} x(t-s)\,ds} - y(t)\right]. \end{aligned} \tag{3.1}$$

Corresponding to system (1.1), one has

$$r_1 = r_2 = \alpha_1 = \alpha_2 = 1, \qquad K_1 = K_2 = 2.$$

By a simple computation, system (3.1) admits a unique positive equilibrium $(\sqrt{2}, \sqrt{2})$. It follows from Theorem 1.1 that $(\sqrt{2}, \sqrt{2})$ is globally attractive. Figures 1 and 2 also support this assertion.

Example 3.2

$$\begin{aligned} \frac{dx(t)}{dt} &= x(t)\left[\frac{3 + 3\int_0^\infty e^{-3s} y(t-s)\,ds}{1 + \int_0^\infty e^{-s} y(t-s)\,ds} - x(t)\right], \\ \frac{dy(t)}{dt} &= y(t)\left[\frac{3 + 3\int_0^\infty e^{-3s} x(t-s)\,ds}{1 + \int_0^\infty e^{-s} x(t-s)\,ds} - y(t)\right]. \end{aligned} \tag{3.2}$$

Corresponding to system (1.1), one has

$$r_1 = r_2 = \alpha_1 = \alpha_2 = 1, \qquad K_1 = K_2 = 3.$$

By a simple computation, system (3.2) admits a unique positive equilibrium $(\sqrt{3}, \sqrt{3})$. It follows from Theorem 1.1 that $(\sqrt{3}, \sqrt{3})$ is globally attractive. Figures 3 and 4 also support this assertion.

Now let us consider the strong integral kernel case.

Figure 2 Dynamic behavior of the second species in system (3.1) with the initial conditions $(x(s), y(s)) = (0.5, 0.5)$, $(0.1, 1)$, $(2, 2)$, $(2.5, 2.5)$ and $(3, 3)$, $s \in (-\infty, 0]$, respectively.

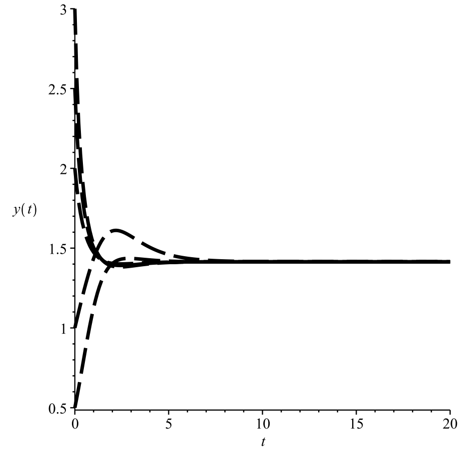


Figure 3 Dynamic behavior of the first species in system (3.2) with the initial conditions $(x(s), y(s)) = (0.5, 0.5)$, $(0.1, 1)$, $(2, 2)$, $(2.5, 2.5)$ and $(3, 3)$, $s \in (-\infty, 0]$, respectively.

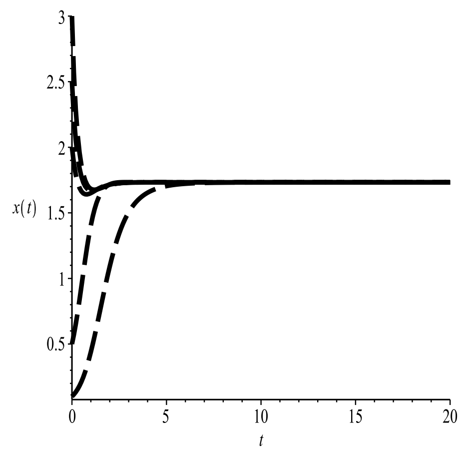


Figure 4 Dynamic behavior of the second species in system (3.2) with the initial conditions $(x(s), y(s)) = (0.5, 0.5)$, $(0.1, 1)$, $(2, 2)$, $(2.5, 2.5)$ and $(3, 3)$, $s \in (-\infty, 0]$, respectively.

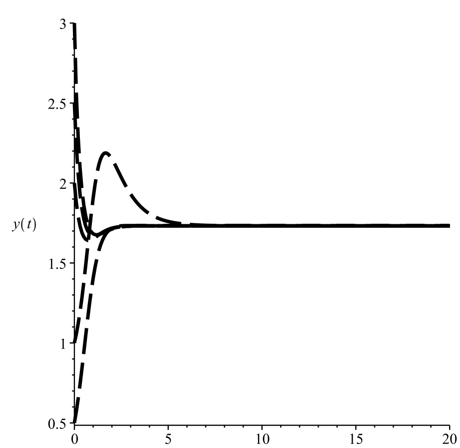


Figure 5 Dynamic behavior of the first species in system (3.3) with the initial conditions $(x(s), y(s)) = (0.5, 0.5), (0.1, 1), (2, 2), (2.5, 2.5)$ and $(3, 3)$, $s \in (-\infty, 0]$, respectively.

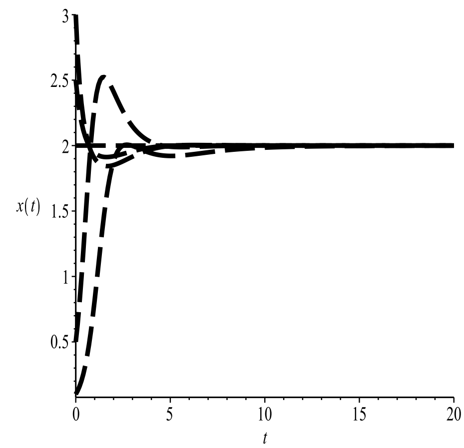
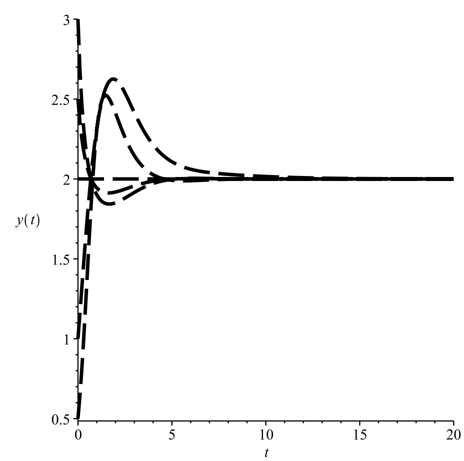


Figure 6 Dynamic behavior of the second species in system (3.3) with the initial conditions $(x(s), y(s)) = (0.5, 0.5), (0.1, 1), (2, 2), (2.5, 2.5)$ and $(3, 3)$, $s \in (-\infty, 0]$, respectively.



Example 3.3

$$\begin{aligned} \frac{dx(t)}{dt} &= x(t)\left[\frac{4 + \int_0^\infty se^{-s}y(t-s)\,ds}{1 + \int_0^\infty e^{-s}y(t-s)\,ds} - x(t)\right], \\ \frac{dy(t)}{dt} &= y(t)\left[\frac{4 + \int_0^\infty se^{-s}x(t-s)\,ds}{1 + \int_0^\infty e^{-s}x(t-s)\,ds} - y(t)\right]. \end{aligned} \tag{3.3}$$

Corresponding to system (1.1), one has

$$r_1 = r_2 = \alpha_1 = \alpha_2 = 1, \qquad K_1 = K_2 = 4.$$

By a simple computation, system (3.3) admits a unique positive equilibrium $(2, 2)$. It follows from Theorem 1.1 that $(2, 2)$ is globally attractive. Figures 5 and 6 also support this assertion.

4 Discussion

Xie et al. [14] studied the stability property of the integro-differential model of mutualism. Under the assumption $\alpha_i > K_i$, $i = 1, 2$, by using the iterative technique, they showed that the system admits a unique globally attractive positive equilibrium. In this paper, we focus our attention on the case $\alpha_i < K_i$, $i = 1, 2$. We first show that this case represents a population system of competition type. Then, by applying the iterative technique, we also show that the system admits a unique globally attractive positive equilibrium.

It is well known that the competitive exclusion principle is the most important rule for a population system. It says that two species competing for the same resource cannot coexist at constant population values. However, as we can see from Theorem 1.1, for all of the parameters which satisfy $K_i > \alpha_i$, $i = 1, 2$, two species could coexist in a stable state. Also, numeric simulations (Figures 1-6) support this assertion. Why did this phenomenon happen? Maybe the reason relies on the term

$$\frac{(K_1 - \alpha_1)\int_0^\infty J_2(s)N_2(t-s)\,ds}{1 + \int_0^\infty J_2(s)N_2(t-s)\,ds} \quad \text{and} \quad \frac{(K_2 - \alpha_2)\int_0^\infty J_1(s)N_1(t-s)\,ds}{1 + \int_0^\infty J_1(s)N_1(t-s)\,ds},$$

since $K_1 > K_1 - \alpha_1$, $K_2 > K_2 - \alpha_2$ and

$$\frac{\int_0^\infty J_2(s)N_2(t-s)\,ds}{1 + \int_0^\infty J_2(s)N_2(t-s)\,ds} < 1, \qquad \frac{\int_0^\infty J_1(s)N_1(t-s)\,ds}{1 + \int_0^\infty J_1(s)N_1(t-s)\,ds} < 1.$$

One could see that the influence of interspecific competition is less than the influence of intrinsic competition, and this leads to stable coexistence of the two species.

Competing interests
The author declares that there is no conflict of interests.

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