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Value distribution of difference and q-difference polynomials

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Abstract

In this paper, we investigate the value distribution of difference polynomial and obtain the following result, which improves a recent result of K. Liu and L.Z. Yang: Let f be a transcendental meromorphic function of finite order σ, c be a nonzero constant, and $\alpha(z) \not\equiv 0$ be a small function of f, and let

$$P(z) = a_n z^n + a_{n-1} z^{n-1} + \cdots + a_1 z + a_0$$

be a polynomial with a multiple zero. If $\lambda(1/f) < \sigma$, then $P(f)f(z+c) - \alpha(z)$ has infinitely many zeros. We also obtain a result concerning the value distribution of q-difference polynomial.

MSC: Primary 30D35; secondary 39A05

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1 Introduction and main results

Throughout the paper, we assume that the reader is familiar with the standard symbols and fundamental results of Nevanlinna theory as found in [1–3]. A function $f(z)$ is called the meromorphic function, if it is analytic in the complex plane except at isolated poles. For any non-constant meromorphic function f, we denote by $S(r,f)$ any quantity satisfying

$$\lim_{r\to\infty} \frac{S(r,f)}{T(r,f)} = 0,$$

possibly outside of a set of finite linear measure in $\mathbb{R}^+$. A meromorphic function $a(z)$ is called a small function of $f(z)$ provided that $T(r,a) = S(r,f)$. As usual, we denote by $\sigma(f)$ the order of a meromorphic function $f(z)$, and denote by $\lambda(f)$ ($\lambda(1/f)$) the exponent of convergence of the zeros (poles) of $f(z)$.

Recently, a number of papers concerning the complex difference products and the differences analogues of Nevanlinna's theory have been published (see [4–12] for example), and many excellent results have been obtained. In 2007, Laine and Yang [10] investigated the value distribution of difference products of entire functions, and obtained the following result.

Theorem A *Let $f(z)$ be a transcendental entire function of finite order, and c be a non-zero complex constant. Then for $n \geq 2$, $f(z)^n f(z+c)$ assumes every non-zero value $a \in \mathbb{C}$ infinitely often.*

Liu and Yang [11] improved Theorem A, and proved the next result.

Theorem B *Let $f(z)$ be a transcendental entire function of finite order, and c be a non-zero complex constant. Then for $n \geq 2$, $f(z)^n f(z+c) - p(z)$ has infinitely many zeros, where $p(z) \not\equiv 0$ is a polynomial in z.*

The purpose of this paper is to investigate the value distribution of difference polynomial $P(f)f(z+c) - \alpha(z)$ and q-difference polynomial $P(f)f(qz) - \alpha(z)$, where $P(z) = a_n z^n + a_{n-1} z^{n-1} + \cdots + a_1 z + a_0$ with constant coefficients $a_n\ (\neq 0), a_{n-1}, \ldots, a_0$, and $\alpha(z)$ is a mall function of $f(z)$.

For the sake of simplicity, we denote by $s(P)$ and $m(P)$ the number of the simple zeros and the number of multiple zeros of a polynomial

$$P(z) = a_n z^n + a_{n-1} z^{n-1} + \cdots + a_1 z + a_0$$

respectively.

We obtain the following result which improves Theorem A and Theorem B.

Theorem 1.1 *Let f be a transcendental meromorphic function of finite order $\sigma(f) = \sigma$, and c be a non-zero constant, and let*

$$P(z) = a_n z^n + a_{n-1} z^{n-1} + \cdots + a_1 z + a_0$$

be a polynomial with constant coefficients $a_n\ (\neq 0), a_{n-1}, \ldots, a_0$ and $m(P) > 0$. If $\lambda(\frac{1}{f}) < \sigma$, then $P(f)f(z+c) - \alpha(z)$ has infinitely many zeros, where $\alpha(z) \not\equiv 0$ is a small function of f.

Remark 1 The result of Theorem 1.1 may be false if $\alpha(z) \equiv 0$, for example, $f(z) = \frac{e^{z^2}}{z}$, it is obvious that $f^2 f(z+1)$ has no zeros. The following example shows that the assumption $\lambda(\frac{1}{f}) < \sigma$ in Theorem 1.1 cannot be deleted. In fact, let $f(z) = \frac{1-e^z}{1+e^z}$, $c = \pi i$, $\alpha(z) = -1$, and $P(z) = z^2$. Then $\lambda(\frac{1}{f}) = \sigma(f) = 1$ and $P(f)f(z+c) - \alpha(z) = \frac{2}{1+e^z}$ has no zeros. Also, let $f(z) = i + e^z$, $c = \pi i$, $\alpha(z) = 1$, and $P(z) = z(z-i+1)(z-i-1)$. Then $P(f)f(z+c) - \alpha(z) = -e^{4z}$ has no zeros. This shows that the restriction in Theorem 1.1 to the multiple zero case is essential.

Considering the value distribution of q-differences polynomials, we obtain the following result.

Theorem 1.2 *Let $f(z)$ be a transcendental entire function of zero order, and $\alpha(z) \in S(r,f)$. Suppose that q is a non-zero complex constant and n is an integer. If $m(P) > 0$, then $P(f)f(qz) - \alpha(z)$ has infinitely many zeros.*

2 Some lemmas

Lemma 2.1 [6] *Given two distinct complex constants η_1, η_2, let f be a meromorphic function of finite order σ. Then, for each $\varepsilon > 0$, we have*

$$m\left(r, \frac{f(z+\eta_1)}{f(z+\eta_2)}\right) = O\left(r^{\sigma-1+\varepsilon}\right).$$

Lemma 2.2 [6] *Let f be a transcendental meromorphic function of finite order σ, c be a complex number. Then, for each $\varepsilon > 0$, we have*

$$T\left(r, f(z+c)\right) = T\left(r, f(z)\right) + O\left(r^{\sigma-1+\varepsilon}\right) + O(\log r).$$

The following lemma is a revised form of Lemma 2.4.2 in [2].

Lemma 2.3 *Let $f(z)$ be a transcendental meromorphic solution of*

$$f^n A(z,f) = B(z,f),$$

where $A(z,f)$, $B(z,f)$ are differential polynomials in f and its derivatives with meromorphic coefficients, say $\{a_\lambda \mid \lambda \in I\}$, n be a positive integer. If the total degree of $B(z,f)$ as a polynomial in f and its derivatives is less than or equal to n, then

$$m\left(r, A(z,f)\right) \le \sum_{\lambda \in I} m(r, a_\lambda) + S(r,f).$$

Lemma 2.4 [12] *Let $f(z)$ be a non-constant meromorphic function of finite order, $c \in \mathbb{C}$. Then*

$$N\left(r, \frac{1}{f(z+c)}\right) \le N\left(r, \frac{1}{f(z)}\right) + S(r,f), \qquad N\left(r, f(z+c)\right) \le N(r,f) + S(r,f),$$
$$\overline{N}\left(r, \frac{1}{f(z+c)}\right) \le \overline{N}\left(r, \frac{1}{f(z)}\right) + S(r,f), \qquad \overline{N}\left(r, f(z+c)\right) \le \overline{N}(r,f) + S(r,f),$$

outside of a possible exceptional set E with finite logarithmic measure.

Lemma 2.5 [4] *Let f be a non-constant zero-order meromorphic function, and $q \in \mathbb{C} \setminus \{0\}$. Then*

$$m\left(r, \frac{f(qz)}{f(z)}\right) = o\left(T(r,f)\right)$$

on a set of logarithmic density 1.

Remark 2 For the similar reason in Theorem 1.1 in [4], we can easily deduce that

$$m\left(r, \frac{f(z)}{f(qz)}\right) = o\left(T(r,f)\right)$$

also holds on a set of logarithmic density 1.

Proof Using the identity

$$\frac{\rho^2 - r^2}{\rho^2 - 2\rho r\cos(\varphi - \theta) + r^2} = \operatorname{Re}\left(\frac{\rho e^{i\theta} + z}{\rho e^{i\theta} - z}\right),$$

and let Poisson-Jensen formula with $R = \rho$, we see

$$\begin{aligned}
\log\left|\frac{f(z)}{f(qz)}\right| &= \int_0^{2\pi} \log\left|f(\rho e^{i\theta})\right| \operatorname{Re}\left(\frac{\rho e^{i\theta} + z}{\rho e^{i\theta} - z} - \frac{\rho e^{i\theta} + qz}{\rho e^{i\theta} - qz}\right)\frac{d\theta}{2\pi} \\
&\quad + \sum_{|a_n|<\rho} \log\left|\frac{(z - a_n)(\rho^2 - \overline{a}_n qz)}{(qz - a_n)(\rho^2 - \overline{a}_n z)}\right| \\
&\quad - \sum_{|b_m|<\rho} \log\left|\frac{(z - b_m)(\rho^2 - \overline{b}_m qz)}{(qz - b_m)(\rho^2 - \overline{b}_m z)}\right| \\
&= S_1'(z) + S_1'(z) - S_3'(z),
\end{aligned}$$

where $\{a_n\}$ and $\{b_m\}$ are the zeros and poles of f, respectively. Integration on the set $E := \{\varphi \in [0, 2\pi] : |\frac{f(re^{i\varphi})}{f(qre^{i\varphi})}| \geq 1\}$ gives us the proximity function,

$$\begin{aligned}
m\left(r, \frac{f(z)}{f(qz)}\right) &= \int_E \log\left|\frac{f(z)}{f(qz)}\right| \frac{d\psi}{2\pi} \\
&= \int_E \left(S_1'(re^{i\psi}) + S_2'(re^{i\psi}) - S_3'(re^{i\psi})\right)\frac{d\psi}{2\pi} \\
&\leq \int_0^{2\pi} \left(\left|S_1'(re^{i\psi})\right| + \left|S_2'(re^{i\psi})\right| + \left|S_3'(re^{i\psi})\right|\right)\frac{d\psi}{2\pi}.
\end{aligned}$$

Since $S_i' = -S_i$ $(i = 1, 2, 3)$ in [4], we get $|S_i'| = |S_i|$ $(i = 1, 2, 3)$.

Following the similar method in the proof of Theorem 1.1 in [4], we get the result. □

Lemma 2.6 *Let f be a non-constant zero-order entire function, and $q \in \mathbb{C} \setminus \{0\}$. Then*

$$T\bigl(r, P(f)f(qz)\bigr) = T\bigl(r, P(f)f(z)\bigr) + S(r, f)$$

on a set of logarithmic density 1.

Proof Since f is an entire function of zero-order, we deduce from Lemma 2.5 that

$$\begin{aligned}
T\bigl(r, P(f)f(qz)\bigr) &= m\bigl(r, P(f)f(qz)\bigr) \\
&\leq m\bigl(r, P(f)f(z)\bigr) + m\left(r, \frac{f(qz)}{f(z)}\right) \\
&\leq m\bigl(r, P(f)f(z)\bigr) + S(r, f) \\
&= T\bigl(r, P(f)f(z)\bigr) + S(r, f),
\end{aligned}$$

that is

$$T\bigl(r, P(f)f(qz)\bigr) \leq T\bigl(r, P(f)f(z)\bigr) + S(r, f). \tag{2.1}$$

On the other hand, using Remark 2, we get

$$\begin{aligned} T\bigl(r,P(f)f(z)\bigr) &= m\bigl(r,P(f)f(z)\bigr) \\ &\le m\bigl(r,P(f)f(qz)\bigr) + m\left(r,\frac{f(z)}{f(qz)}\right) \\ &\le m\bigl(r,P(f)f(qz)\bigr) + S(r,f) \\ &= T\bigl(r,P(f)f(qz)\bigr) + S(r,f), \end{aligned}$$

that is

$$T\bigl(r,P(f)f(z)\bigr) \le T\bigl(r,P(f)f(qz)\bigr) + S(r,f). \tag{2.2}$$

The assertion follows from (2.1) and (2.2). □

3 Proof of Theorem 1.1

Let $\beta(z)$ be the canonical products of the nonzero poles of $P(f)f(z+c)-\alpha(z)$. Since $\lambda(1/f) < \sigma$ and $\alpha(z)$ is a small function of $f(z)$, we know that $\sigma(\beta) = \lambda(\beta) < \sigma(f)$. Suppose on contrary to the assertion that $P(f)f(z+c)-\alpha(z)$ has finitely many zeros. Then we have

$$P(f)f(z+c) - \alpha(z) = R(z)e^{Q(z)}/\beta(z),$$

where $Q(z)$ is a polynomial, and $R(z) \not\equiv 0$ is a rational function. Set $H(z) = R(z)/\beta(z)$. Then

$$\sigma(H) < \sigma(f) = \sigma, \tag{3.1}$$

and

$$P(f)f(z+c) - \alpha(z) = H(z)e^{Q(z)}. \tag{3.2}$$

Differentiating (3.2) and eliminating $e^{Q(z)}$, we obtain

$$\begin{aligned} &P'(f)f'(z)f(z+c)H(z) + P(f)f'(z+c)H(z) - P(f)f(z+c)H'(z) - P(f)f(z+c)Q'(z)H(z) \\ &\quad = \alpha'(z)H(z) - \alpha(z)H'(z) - \alpha(z)Q'(z)H(z). \end{aligned} \tag{3.3}$$

Let $\alpha_1, \alpha_2, \ldots, \alpha_t$ be the distinct zeros of $P(z)$. Then

$$P(f) = a_n(f-\alpha_1)^{n_1}(f-\alpha_2)^{n_2}\cdots(f-\alpha_t)^{n_t}.$$

Substituting this into (3.3), we have

$$\begin{aligned} &a_n\prod_{j=1}^{t}(f-\alpha_j)^{n_j-1}\left\{\left(n_1\prod_{j\ne 1}(f-\alpha_j) + n_2\prod_{j\ne 2}(f-\alpha_j) + \cdots + n_t\prod_{j\ne t}(f-\alpha_j)\right)\right. \\ &\quad \times f(z+c)H(z)f'(z) + f'(z+c)H(z) \end{aligned}$$

$$\begin{aligned}&\times\prod_{j=1}^{t}(f-\alpha_j)-f(z+c)\bigl(H'(z)+Q'(z)H(z)\bigr)\prod_{j=1}^{t}(f-\alpha_j)\Biggr\}\\&=\alpha'(z)H(z)-\alpha(z)H'(z)-\alpha(z)Q'(z)H(z).\end{aligned}$$

Note that $P(z)$ has at least one multiple zero, we may assume that $n_1 > 1$ without loss of generality, and we have

$$a_n(f-\alpha_1)^{n_1-1}F(z,f)=\alpha'(z)H(z)-\alpha(z)H'(z)-\alpha(z)Q'(z)H(z), \tag{3.4}$$

where

$$\begin{aligned}F(z,f)={}&\prod_{j=2}^{t}(f-\alpha_j)^{n_j-1}\Biggl\{\biggl(n_1\prod_{j\neq 1}(f-\alpha_j)+n_2\prod_{j\neq 2}(f-\alpha_j)+\cdots+n_t\prod_{j\neq t}(f-\alpha_j)\biggr)\\&\times f(z+c)H(z)f'(z)+f'(z+c)H(z)\prod_{j=1}^{t}(f-\alpha_j)\\&-f(z+c)\bigl(H'(z)+Q'(z)H(z)\bigr)\prod_{j=1}^{t}(f-\alpha_j)\Biggr\}.\end{aligned}$$

Now we distinguish two cases.

Case 1. $F(z,f)\equiv 0$. In this case, we obtain from (3.4) that

$$\alpha'(z)H(z)-\alpha(z)H'(z)-\alpha(z)Q'(z)H(z)\equiv 0.$$

Since $\alpha(z)\not\equiv 0$ and $H(z)\not\equiv 0$, by integrating, we have

$$\frac{\alpha(z)}{H(z)}=ke^{Q(z)}, \tag{3.5}$$

where k is a non-zero constant. From (3.2) and (3.5), we have

$$P(f)f(z+c)=\left(\frac{1}{k}+1\right)\alpha(z).$$

By Lemma 2.2, we have

$$\begin{aligned}nT\bigl(r,f(z)\bigr)&=T\bigl(r,P(f)\bigr)+O(1)\\&\le T\bigl(r,f(z+c)\bigr)+T\bigl(r,\alpha(z)\bigr)+O(1)\\&=T\bigl(r,f(z)\bigr)+O\bigl(r^{\sigma-1+\varepsilon}\bigr)+S(r,f).\end{aligned}$$

Since $n\ge n_1\ge 2$, and $f(z)$ is a transcendental, this is impossible.

Case 2. $F(z,f) \not\equiv 0$. In this case, we set

$$\begin{aligned}
F^{*}(z,f) &= \frac{F(z,f)}{f-\alpha_1} \\
&= \prod_{j=2}^{t}(f-\alpha_j)^{n_j-1}\Bigg\{\bigg(n_1\prod_{j\neq 1}(f-\alpha_j)+n_2\prod_{j\neq 2}(f-\alpha_j)+\cdots+n_t\prod_{j\neq t}(f-\alpha_j)\bigg) \\
&\quad \times \frac{f(z+c)}{f(z)}f(z)H(z)\frac{f'(z)}{f-\alpha_1}+\frac{f'(z+c)}{f(z+c)}\frac{f(z+c)}{f(z)}f(z)H(z)\prod_{j=2}^{t}(f-\alpha_j) \\
&\quad -\frac{f(z+c)}{f(z)}f(z)\big(H'(z)+Q'(z)H(z)\big)\prod_{j=2}^{t}(f-\alpha_j)\Bigg\}.
\end{aligned}$$

Since $f(z) = (f(z) - \alpha_1) + \alpha_1$ and $f^{(k)} = (f - \alpha_1)^{(k)}$, we know that $F^{*}(z,f)$ is a differential polynomial of $f(z) - \alpha_1$ with meromorphic coefficients, and

$$a_n(f-\alpha_1)^{n_1}F^{*}(z,f) = \alpha'(z)H(z) - \alpha(z)H'(z) - \alpha(z)Q'(z)H(z). \tag{3.6}$$

By Lemma 2.3, we have

$$\begin{aligned}
m\big(r,(f-\alpha_1)^k F^{*}(z,f)\big) &\le 3m\left(r,\frac{f(z+c)}{f(z)}\right)+m\left(r,\frac{f'(z+c)}{f(z+c)}\right)+m\left(r,\frac{f'(z)}{f-\alpha_1}\right) \\
&\quad + 5T(r,H)+S(r,f)
\end{aligned} \tag{3.7}$$

for $k = 0$ and $k = 1$.

Now for any given ε $(0 < \varepsilon < 1)$, we obtain from Lemma 2.1, Lemma 2.2 and (3.1) that

$$m\left(r,\frac{f(z+c)}{f(z)}\right) = O\big(r^{\sigma-\varepsilon}\big), \qquad T(r,H) = O\big(r^{\sigma-\varepsilon}\big), \tag{3.8}$$

$$m\left(r,\frac{f'(z+c)}{f(z+c)}\right) = O\big(r^{\sigma-\varepsilon}\big) + S(r,f). \tag{3.9}$$

The lemma of logarithmic derivative implies that

$$m\left(r,\frac{f'(z)}{f-\alpha_1}\right) = S(r,f). \tag{3.10}$$

It follows from (3.7) to (3.10) that

$$m\big(r,F^{*}(z,f)\big) = O\big(r^{\sigma-\varepsilon}\big) + S(r,f), \tag{3.11}$$

$$m\big(r,(f-\alpha_1)F^{*}(z,f)\big) = O\big(r^{\sigma-\varepsilon}\big) + S(r,f). \tag{3.12}$$

Since $(f - \alpha_1)F^{*}(z,f) = F(z,f)$, we obtain from the definition of $F(z,f)$ that

$$N\big(r,F(z,f)\big) = O\big(N\big(r,H(z)\big) + N(r,f)\big) = O\big(r^{\sigma-\varepsilon}\big) + S(r,f).$$

Thus,

$$T\big(r,(f-\alpha_1)F^*(z,f)\big)=O\big(r^{\sigma-\varepsilon}\big)+S(r,f). \tag{3.13}$$

Note that, a zero of $f(z)-\alpha_1$ which is not a pole of $f(z+c)$ and $H(z)$, is a pole of $F^*(z,f)$ with the multiplicity at most 1, we know from (3.6), (3.1), Lemma 2.4 and $\lambda(1/f)<\sigma$ that

$$\begin{aligned}(n_1-1)N\left(r,\frac{1}{f(z)-\alpha_1}\right)&\le N\left(r,\frac{1}{\alpha'(z)H(z)-\alpha(z)H'(z)-\alpha(z)Q'(z)H(z)}\right)\\&\quad+O\big(N\big(r,f(z+c)\big)\big)+O\big(N(r,H)\big)\\&=O\big(r^{\sigma-\varepsilon}\big)\end{aligned} \tag{3.14}$$

for the positive ε sufficiently small. Hence (see the definition of $F^*(z,f)$),

$$\begin{aligned}N\big(r,F^*(z,f)\big)&=O\left(N\left(r,\frac{1}{f-\alpha_1}\right)+N(r,f)+N(r,H)\right)\\&=O\big(r^{\sigma-\varepsilon}\big)+S(r,f).\end{aligned} \tag{3.15}$$

It follows from (3.15) and (3.11) that

$$T\big(r,F^*(z,f)\big)=O\big(r^{\sigma-\varepsilon}\big)+S(r,f). \tag{3.16}$$

Thus, we deduce from (3.16) and (3.13) that

$$\begin{aligned}T\big(r,f(z)\big)&=T\big(r,f(z)-\alpha_1\big)+O(1)=T\left(r,\frac{(f-\alpha_1)F^*(z,f)}{F^*(z,f)}\right)\\&=O\big(r^{\sigma-\varepsilon}\big)+S(r,f).\end{aligned}$$

This contradicts that f is of order σ. Theorem 1.1 is proved.

4 Proof of Theorem 1.2

Denote $F(z)=P(f)f(qz)$. From Lemma 2.6 and the standard Valiron-Mohon'ko theorem, we deduce

$$\begin{aligned}T\big(r,F(z)\big)&=T\big(r,P(f)f(z)\big)+S(r,f)\\&=(n+1)T\big(r,f(z)\big)+S(r,f).\end{aligned}$$

Since f is a entire function, then by the second main theorem and Lemma 2.5, we have

$$\begin{aligned}T\big(r,F(z)\big)&\le\overline{N}\big(r,F(z)\big)+\overline{N}\left(r,\frac{1}{F(z)}\right)+\overline{N}\left(r,\frac{1}{F(z)-\alpha(z)}\right)+S(r,f)\\&\le\overline{N}\left(r,\frac{1}{P(f)}\right)+\overline{N}\left(r,\frac{1}{f(qz)}\right)+\overline{N}\left(r,\frac{1}{F(z)-\alpha(z)}\right)+S(r,f)\\&\le\big(s(P)+m(P)\big)T\big(r,f(z)\big)+T\big(r,f(qz)\big)\\&\quad+\overline{N}\left(r,\frac{1}{F(z)-\alpha(z)}\right)+S(r,f)\end{aligned}$$

$$\begin{aligned}
&\le \big(s(P)+m(P)\big)T\big(r,f(z)\big)+m\left(r,\frac{f(qz)}{f(z)}\right)+m\big(r,f(z)\big)\\
&\quad +\overline{N}\left(r,\frac{1}{F(z)-\alpha(z)}\right)+S(r,f)\\
&\le \big(s(P)+m(P)+1\big)T\big(r,f(z)\big)+\overline{N}\left(r,\frac{1}{F(z)-\alpha(z)}\right)+S(r,f),
\end{aligned}$$

that is,

$$\overline{N}\left(r,\frac{1}{F(z)-\alpha(z)}\right)\ge \big(n-s(P)-m(P)\big)T\big(r,f(z)\big)+S\big(r,f(z)\big).$$

Since f is a transcendental entire function with $m(P)>0$, we deduce that $P(f)f(qz)-\alpha(z)$ has infinitely many zeros.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors drafted the manuscript, read and approved the final manuscript.

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